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THE UNIVERSITY OF ALBERTA

CONDUCTIVE STRUCTURES IN THE LITHOSPHERE OF WESTERN CANADA

by

GUANGMING CHEN



A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

IN

GEOPHYSICS

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EDMONTON, ALBERTA

FALL 1987

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled CONDUCTIVE STRUCTURES IN THE LITHOSPHERE OF WESTERN CANADA submitted by GUANGMING CHEN in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in GEOPHYSICS.

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Date...October 8, 1987.....

## ABSTRACT

This investigation used data from the 1983 and 1981B magnetovariation arrays in western Canada. Both two-dimensional arrays were located in southern Alberta and the south-eastern corner of British Columbia, across the Rocky Mountains, with 33 and 34 Gough-Reitzel magnetometers respectively.

Two sets of magnetograms and ten sets of Fourier transform (FT) maps, covering the period range 8.2-170.6 minutes, are presented. They show the eastern limit of the Canadian Cordilleran Regional (CCR) conductor, and the presence of the Southern Alberta-British Columbia (SABC) conductor.

Single-station and average-array transfer functions have been calculated for bands of periods centred at 8.5, 17.0 and 49.0 minutes. These are derived from 6 to 16 independent magnetovariation events with well distributed polarizations to reduce contamination with the fields of external currents. The induction arrows and hypothetical events give a more accurate location of the SABC conductor than FT maps of simple variation events, and show the Northern Rockies (NR) conductor clearly. The SABC conductor is a very large one. Its central line runs from the Idaho-British Columbia border northeastward across the Rocky Mountains, through Alberta and into Saskatchewan. The SABC conductor does not coincide with the Precambrian rift in southwestern Canada, which was proposed by Kanasewich. The

NR conductor continues to the SABC conductor, and therefore runs at least over 400 km along the Main Ranges of the Rockies from  $54^{\circ}$  to  $50.8^{\circ}\text{N}$ . The eastern limit of the CCR conductor is formed by the NR conductor north of  $50.8^{\circ}\text{N}$ , and runs along the SABC conductor between  $50.8^{\circ}$  and  $49^{\circ}\text{N}$ .

Techniques of separation of internal field from observed magnetovariation fields, and of downward continuation of the internal field, have been applied to the problem of the depth of the SABC conductor. The results show that the upper surface of this conductor lies between 25-30 km below the surface. The conductor thus lies in the lithosphere, mainly below the Riel discontinuity, whereas the Precambrian rift lies above the Riel discontinuity.

The thesis ends with a brief discussion of tectonic features in southwestern Canada and of possible causes of the high conductivity in the SABC conductor.

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## 1. INTRODUCTION

Electrical conductivity is one of the physical properties of rocks constituting the Earth's crust and mantle. Its definition is given by Ohm's law, which is reasonable because of small fields and low frequencies,

$$\vec{J} = \sigma \vec{E} \quad (1.1)$$

where  $\vec{E}$  is the electric field strength in volts per meter;  $\vec{J}$  is the current density in amperes per square meter and  $\sigma$  is known as the electrical conductivity in Siemens per meter, which is a scalar for an isotropic medium or a tensor for an anisotropic one. The electrical conduction of the material is due to movement of electrons or ions in an electric field. The electrons or ions may be free or bound.

Therefore, there are four types of conduction mechanism in the Earth (see Parkinson, 1983): (1) free electron conduction in metals; (2) bound electron conduction in semi-conductors; (3) free ions conduction in electrolytic solutions; and (4) bound ions conduction in ionic bonded crystals.

From studies of conduction mechanisms we can infer that the conductivity will be related to other physical quantities, for example (see Parkinson, 1983),

1. Temperature: In the bound ion conduction, the conductivity depends on absolute temperature (T) by the relation

$$\sigma = \sigma_0 \exp(-E_0/kT) + \sigma_1 \exp(-E_1/kT) \quad (1.2)$$

where  $k$  is Boltzmann's constant. The first term



represents extrinsic conductivity due to crystal defects and impurities, which dominates at low temperatures, and the second term represents intrinsic conductivity due to thermal motion, which dominates at high temperatures. The quantities  $E_e$  and  $E_i$  are the activation energies required to release these ions, and  $\sigma_e$  and  $\sigma_i$  are limiting conductivities as  $T$  goes to  $\infty$ . The relation (1.2) also applies to the bound electron conduction, where  $E_e$  and  $E_i$  represent the gaps between the conduction band and the valence band. In the free ion conduction conductivity also increases with increasing temperature, because the viscosity decreases. Thus in three types of conduction mentioned above the electrical conductivity is highly sensitive to temperature. In principle, one can infer the temperature distribution of the crust and mantle from their conductivity distribution when a conduction mechanism are assumed. The free electron conduction is important only in the core.

2. Phase change: It has been found that the melt phase of a mineral has a much higher conductivity than the solid state (Presnall et al., 1972). Melting of rocks in the upper mantle or in localized areas of the lower crust may increase the electrical conductivity. Some authors have used partial melt to interpret the observed conductivity anomalies (Berdichevsky et al., 1972) and estimated how electrical conductivity varies with the

concentration of partial melt (Waff, 1974; Shankland and Waff, 1977).

3. Water content: For most rocks near the Earth's surface, the conduction is free ionic. The conductivity obeys Archie's law (Archie, 1942)

$$\sigma = a\sigma_s S^m P^n \quad (1.3)$$

where  $\sigma_s$  is the conductivity of the solution;  $S$  the saturation (1.0 for completely saturated rock);  $P$  the porosity; and  $a$ ,  $m$ ,  $n$  are empirically determined parameters which seem to depend on the texture of the rock, for example,  $a=1$  and  $m$ ,  $n$  having values close to 2 in oil reservoir rocks (Keller, 1968). The electrical conductivity of the rock increases with increasing water content.

4. Chemical composition: The electrical conductivity also depends on the chemical compositions of rocks. The electrical conductivities of conductors, semi-conductors and dielectrics cover a very wide range. The electrical conductivities among sedimentary rocks, igneous rocks and metamorphic rocks are very different. Even the two minerals forsterite ( $Mg_2SiO_4$ ) and fayalite ( $Fe_2SiO_4$ ) which make up the olivine in the mantle have different conductivities. The crust contains highly conducting minerals such as sulphides, magnetite and graphite. Concentrations of these minerals may explain some conductivity anomalies.

Therefore, studies of conductive structures in the Earth may

provide information on the phase, chemical composition and thermodynamics and hydrodynamics of the Earth's crust and upper mantle.

The global conductivity within the Earth has been searched for on the basis of a principle of electromagnetic induction by means of naturally occurring long-period time-varying magnetic fields observed at magnetic observatories. The models of radial conductivity distribution has been inferred by various authors. Bailey (1973) and Rikitake (1973) The models of radial conductivity distribution has been The models of radial conductivity distribution has been have given excellent reviews of these models. Parkinson (1983) compiled several important models of radial conductivity distribution on the continent as well as several conductivity profiles determined in the ocean or on ocean islands. Figure 1.1 reproduces his results. These models display a region of low conductivity in top several hundred kilometers, followed by a rapid rise in the depth range 400 to 800 km, to values of 1 S/m or more. Below 700 km the conductivity rises more slowly with depth, but the absorption of the incident fields limits knowledge of conductivity in the middle and lower mantle, where the conductivity may level off. The main difference between oceanic and continental profiles is that conductivity increases at a much shallower depth beneath the ocean.

Since the early 1950's it has been found that in many regions the vertical components of magnetovariation fields

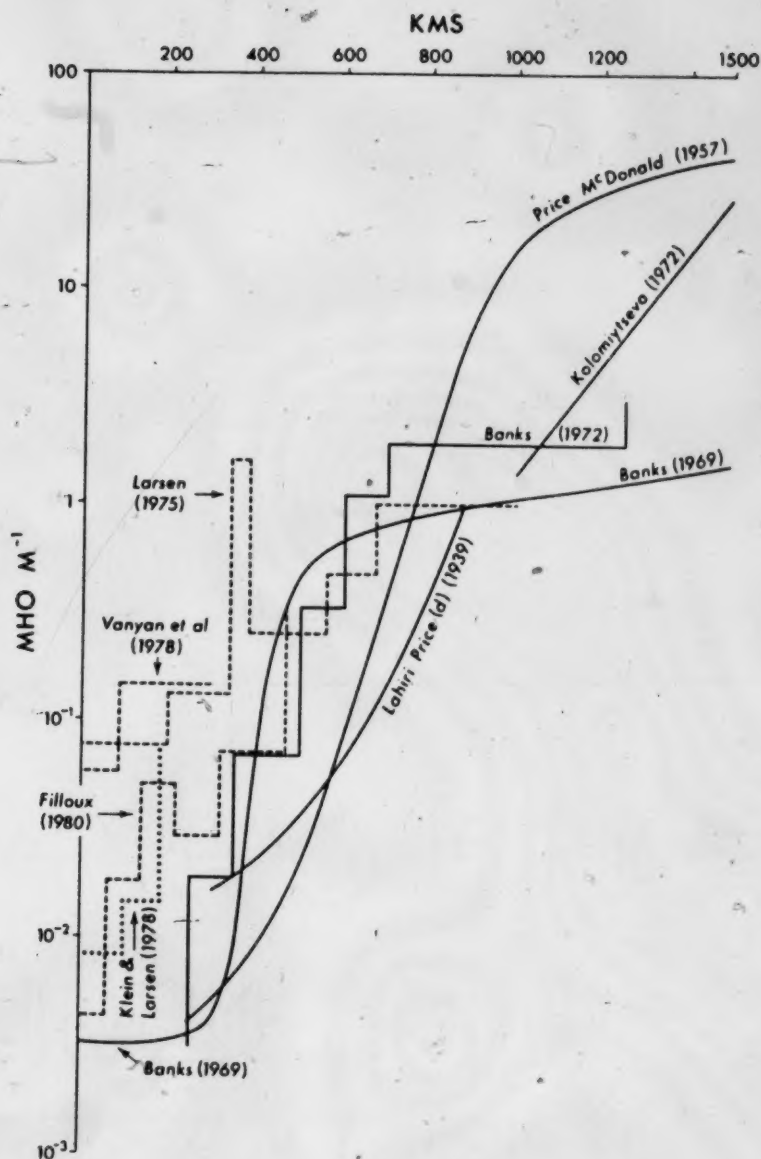


Figure 1.1 Estimates of conductivity as a function of depth. Solid lines refer to continental measurements, dashed or dotted lines to ocean floor or ocean island measurements. Reproduced from Parkinson (1983).



with periods shorter than a few hours vary considerably between sites some tens of kilometers apart. Such special changes of magnetovariation fields can be accounted for neither by an external field nor by induction in a laterally uniform Earth. They can reasonably be attributed to lateral heterogeneity of electrical conductivity in the Earth's crust and upper mantle, which is called a conductivity anomaly.

The most widely applied methods for investigating such conductivity anomalies in the crust and upper mantle have involved measurements of natural electromagnetic fields on the Earth's surface. Two techniques are used. In one, the three orthogonal components of the magnetovariation field are simultaneously recorded at several to several dozen points over the area of interest on the Earth's surface. This technique is known as the magnetovariation array method or sometimes as geomagnetic depth sounding. Magnetovariation studies (MVS) were first developed by Schmucker (1964, 1970), who used up to eight magnetometers in a line across an elongated feature or structure. Such linear arrays were followed by large two-dimensional magnetovariation arrays, with the introduction of a cheap, reliable and very fieldworthy magnetometer designed and constructed by Gough and Reitzel (1967). Magnetovariation array studies involve observation, analysis and interpretation of anomalies of magnetovariation fields at the Earth's surface. Many techniques, such as magnetograms, Fourier transform anomaly

maps, separation of observed total fields into external and internal parts, analyses of transfer functions (representations of induction arrows and hypothetical events), modelling, inversion and so forth, have been developed and used in analyses and interpretations of magnetovariation studies. A recent review of the magnetovariation techniques is given by Gough and Ingham (1983).

The second technique makes use of three orthogonal magnetic components and two horizontal electric components of the natural electromagnetic field simultaneously recorded at a site of the Earth's surface, and is known as the magnetotelluric method or magnetotelluric sounding (MTS). The MTS method was originally proposed by Cagniard (1953) for a uniform source field and a horizontally homogeneous stratified Earth. Price (1962) developed the method for finite source fields. Cantwell (1960) extended the method for anisotropic or two-dimensional structures and found a relationship between the horizontal magnetic and electric fields at the Earth's surface

$$[\vec{E}] = \{Z\}[\vec{H}] \quad (1.4)$$

$$\text{and} \quad \{Z\} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \quad (1.5)$$

where the dyad  $Z$  is termed the impedance tensor, and  $Z_{xy}$ ,  $Z_{yx}$  are the principal impedances and  $Z_{xx}$ ,  $Z_{yy}$  are the additional impedances caused by anisotropy or inhomogeneity.

Swift (1967) investigated the properties of these tensors. For an isotropic layered Earth,

$$Z_{xx} = Z_{yy} = 0 \quad (1.6)$$

$$Z_{xy} = -Z_{yx} \quad (1.7)$$

When the Earth layers contain anisotropy or lateral inhomogeneity of two-dimensional type

$$Z_{xx} = -Z_{yy} \quad (1.8)$$

$$Z_{xy} \neq Z_{yx} \quad (1.9)$$

Sims and Bostick (1969) presented a method to compute tensor impedance elements from observed data.

The impedance tensor may be transformed to apparent resistivity and phase by the following formulas,

$$\rho_{ij} = |Z_{ij}|^2 / \mu\omega \quad (1.10)$$

$$\text{and } \theta_{ij} = \tan^{-1}[\text{Im}(Z_{ij})/\text{Re}(Z_{ij})] \quad i, j = x, y \quad (1.11)$$

where  $\text{Im}(Z_{ij})$  and  $\text{Re}(Z_{ij})$  are the imaginary and real parts of  $Z_{ij}$ , respectively, and where the phase  $\theta_{ij}$  between E and H is the angle measured counterclockwise in the complex plane.

The impedance tensor or apparent resistivity as function of frequency, can be converted to resistivity-depth profiles in terms of inversion and modelling techniques. Inversion methods of Marquardt (Wu, 1968) and Monte-Carlo (Jones and Hutton, 1979) have widely been used. The four basic numerical techniques have been developed to model magnetotelluric responses in 1-D, 2-D and 3-D structures. They are: (1) the finite difference method (Jones and Price 1970; Lines and Jones, 1973a and b); (2) the finite element method (Coggon, 1971; Reddy et al., 1977; Pridmore et al.,

1981); (3) the integral equation method (Dmitriev and Zakharov, 1970; Hohmann, 1975; Weidelt, 1975; Wannamaker et al., 1984); (4) the transmission surface analogy (Madden and Thompson, 1965; Madden and Swift, 1969; Swift, 1967, 1971; Wright, 1969, 1970).

Two modern textbooks (Berdichevsky and Zhdanov, 1984; Kaufman and Keller, 1981) have described the MVS and MTS methods.

The magnetovariation (MVS) and magnetotelluric (MTS) techniques are complementary in the study of conductive structures in the crust and upper mantle of the Earth. MVS acts as a delineator, locating the anomaly and yielding a map of its location. MTS acts as a calibrator, giving estimates of conductivities and depths. The combination of the two techniques makes results more effective than either alone.

### **1.1 Previous Magnetovariation Studies in Western North America**

Magnetovariation studies in western North America were of two kinds: linear and two-dimensional arrays.

#### **1.1.1 Linear array studies**

Linear array studies in western North America were conducted in two areas: the southwestern United States and the western Canadian Cordillera. Pioneering linear array studies were first developed by Schmucker (1964; 1970) in



the southwestern United States. Schmucker, employing six portable Askania Variographs, made geomagnetic observations at 49 locations during the years 1959-1962. Figure 1.2 shows the locations of the 49 stations. The distance between adjacent stations varied between 20 km and 100 km depending on the nature of the conductivity anomaly studied. Schmucker located two major conductivity anomalies of interest. The first, the "California coastal anomaly", is marked by strong variations in the vertical component (Z) of magnetovariation fields along the California coast which gradually diminish inland and were explained as an edge effect of the Pacific Ocean. The second is the "Texas inland anomaly". Schmucker operated an east-west linear array at latitude  $32^{\circ}\text{N}$  from Tucson (Arizona) eastward for about 1200 km to Sweetwater (Texas) (also see Figure 1.3). He observed a sharp discontinuity (a factor of about 3) in the Z variation fields between Las Cruces and Cornudas, New Mexico, at longitude about  $106^{\circ}\text{W}$ , with low amplitudes to the west. The horizontal components H (magnetic N-S) and D (magnetic E-W) of magnetovariation fields were relatively uniform along the array. These characters of magnetovariation field are shown in Figure 1.3, which also shows a reversal in Z between Cornudas and Las Cruces, diagnostic of a current concentration between these stations. Schmucker (1964) interpreted this as being due to a rise in the top surface of the conducting mantle, with a change in depth from 160 km in the west to 320 km under the eastern region. The



Figure 1.2 Locations of 49 stations run by Schmucker during 1959-1962. Modified from Schmucker (1964).

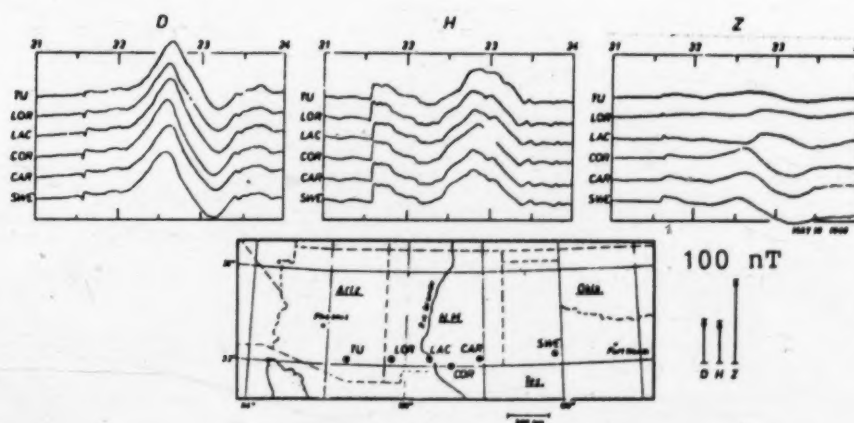


Figure 1.3 Geomagnetic bay recorded by Schmucker at stations in Arizona, New Mexico, and Texas. Modified from Schmucker (1964).

sharpness of the transition between Las Cruces and Cornudas led Schmucker, however, to postulate an additional, more shallow, conductive region between the two stations. A similar pattern was found on the recordings of the temporary IGY stations during 1957 to 1958 around latitude  $40^{\circ}\text{N}$ . The Z/H ratios at three eastern stations (Burlington, Colorado; Beloit, Kansas; and Carrollton, Missouri) were about 3-4 times higher than those recorded at Leadville, Colorado, and Espanola, New Mexico (Schmucker, 1964; Caner et al., 1967). Gough and Reitzel (1969) in 1966 used a linear array with eight Gough-Reitzel (GR) magnetometers along Arkansas and Gunnison Rivers in Colorado between  $38^{\circ}$  and  $39^{\circ}\text{N}$  in order to look for a possible northward continuation of the 'Texas inland anomaly'. Z magnetovariation field again increases by a factor of about 3 from west to east. The transition occurs near Leadville, which is about 100 km west of the front of the Rockies. Caner et al. (1967) observed a linear array about 750 km long near  $35^{\circ}\text{N}$  from Horse Spring (HOK), New Mexico, to Sayre (SAY), Oklahoma; this array was extended another 250 km eastward by a permanent observatory at Norman, Oklahoma. The results showed that the anomaly between Las Cruces and Cornudas, New Mexico reappeared to the north between Bushland (BUS) and Norman. Sayre was a transition station from low-Z Bushland to high-Z Norman. Figure 1.4 shows the locations of the linear array in southwestern United States, as well as the 1967 two-dimensional array.

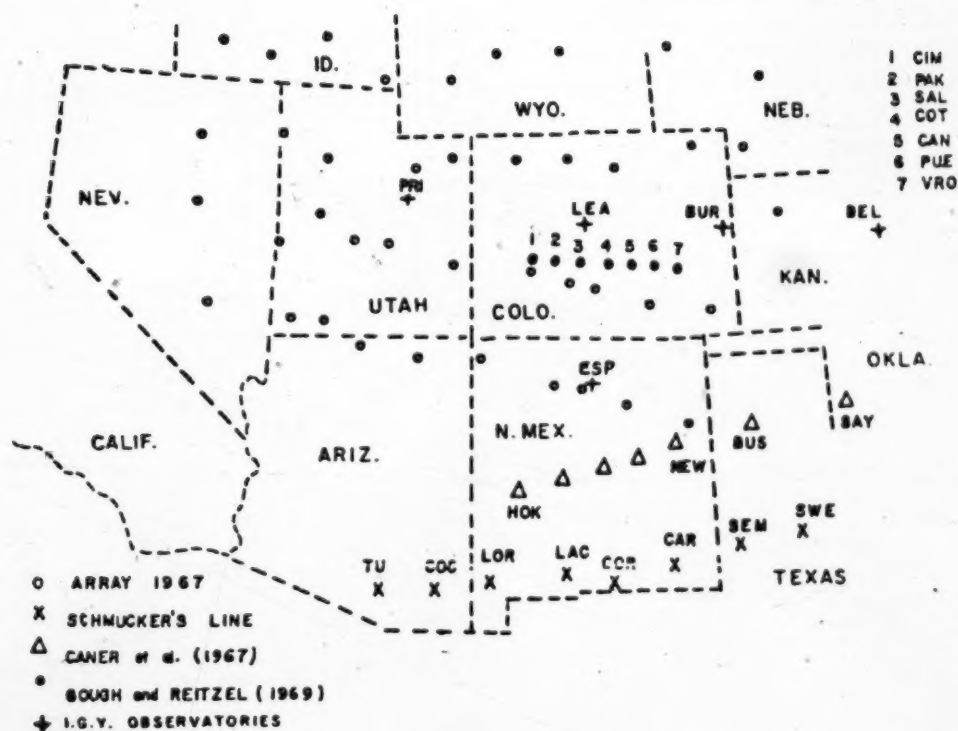


Figure 1.4 Linear arrays in southwestern United States, as well as the 1967 two-dimensional array of Reitzel et al. (1970). From Oldenburg (1969).



During and after the linear array studies in the southwestern United States, Hyndman (1963), and then Caner and his colleagues carried out a series of linear array studies in the western Canadian Cordillera (Lambert and Caner, 1965; Caner et al., 1967; Cochrane and Hyndman, 1970; Dragert, 1970; Lajoie and Caner, 1970; Caner et al., 1971; Dragert and Clarke, 1977). Figure 1.5 shows the locations of these linear arrays. Stations are coded according to their I-value, which is defined as

$$I = \frac{|\Delta Z|}{[(\Delta D)^2 + (\Delta H)^2]^{1/2}} \quad (1.12)$$

We will discuss the results of these array studies in detail later.

### 1.1.2 Two-dimensional array studies

The two-dimensional magnetovariation array studies may also be grouped in two areas: western United States and western Canada.

During 1967-1969 three large two-dimensional magnetovariation arrays were run in western North America between the Coastal Ranges and the Great Plains, and from the Mexico-United States border to the Trans-Canada Highway, in order to study the conductive structure of the Earth's crust and upper mantle. These array studies were jointly undertaken by the University of Alberta and the University of Texas at Dallas. Their locations are shown in Figure 1.6 (Camfield, 1973). The array of 1967, with 42 Gough-Reitzel



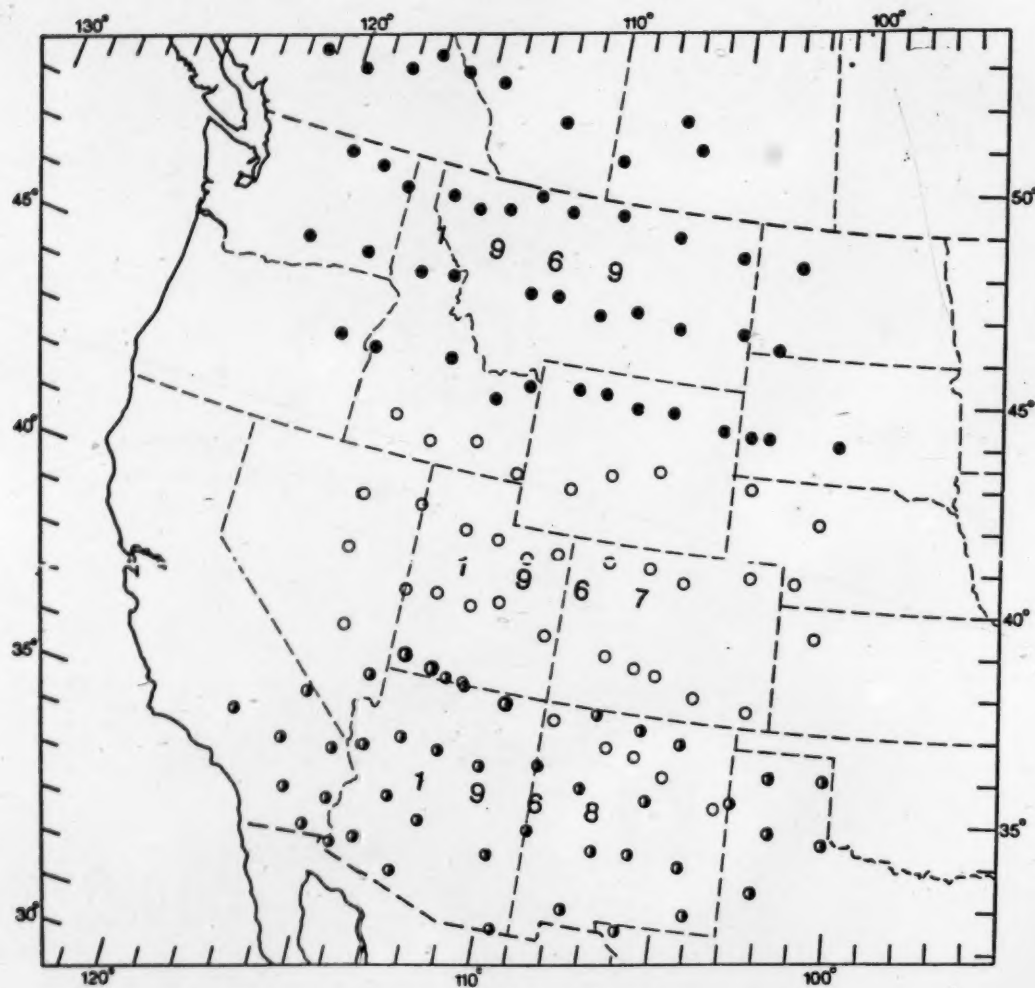


Figure 1.6 Location of three large two-dimensional magnetovariation arrays in western North America during 1967-1969. Open circles, 1967 array; half-filled circles, 1968 array; filled circles, 1969. From Camfield (1973).

(GR) magnetometers, was operated between latitudes  $36^{\circ}$  and  $43^{\circ}$ N and longitudes  $101^{\circ}$  and  $116^{\circ}$ W (Reitzel et al. 1970; Porath et al., 1970; Porath and Gough, 1971; Porath et al., 1971) and was designed to map an anomaly previously discovered by Gough and Reitzel (1969) in Colorado. The results of the array study indicated two anomalies: a ridge of high conductivity runs at a depth no greater than 200 km under the Southern Rocky Mountains between the Great Plains and Colorado Plateau; the other strong conductivity anomaly runs north-south along the Wasatch Front through central Utah at the boundary between the Basin and Range province and the Colorado plateau, the depth of which is not greater than 120 km. The 1968 array (Porath and Gough, 1971) was operated in the southwestern United States in order to look for a southward continuation of the magnetovariation field anomalies found by the array of 1967. This array covered the area from western Texas across New Mexico and Arizona into California. It overlapped the 1967 array, and included the linear array made by Schmucker (1964) across-southern Arizona, New Mexico and western Texas. Analyses of magnetograms and Fourier transform (FT) maps showed that conductive structures under the Southern Rockies and Wasatch Fault Belt, discovered by the array study of 1967, continued southward. The southern end of the Wasatch Front anomaly becomes merged in the general high conductivity of the mantle beneath the Basin and Ranges. The 1969 array of 46 stations was operated in the north-western United States and



south-western Canada between latitudes  $44^{\circ}$  and  $51^{\circ}$ N and longitudes  $100^{\circ}$  and  $121^{\circ}$ W (Camfield et al. 1971). The instruments were 43 GR magnetometers, three fluxgate magnetometers and one standard-run Ruska magnetometer.

This array study completed the reconnaissance survey of magnetic variation anomalies in western North America from the U.S.-Mexico border to the trans-Canada Highway. It linked the linear array by Schmucker (1964) in southwestern United States to those by Hyndman (1963) and Caner, Cannon and Livingston (1967) across southern British Columbia. Analyses of magnetograms and FT maps, and two-dimensional modelling revealed the North American Central Plains Anomaly (NACP) running from the eastern edge of the Black Hills northward along the boundary between Montana and the Dakotas to the Williston Basin and crossing the USA-Canada border (Camfield et al., 1971; Porath et al., 1971; Camfield, 1973; Camfield and Gough, 1975). Its small half-width indicates that it is a crustal conductor. In the western and southern parts of the 1969 array, the main feature of the magnetovariation fields is a southwestward decrease in Z component. This decrease was attributed to a rise of a good conductor in the upper mantle or lower crust to shallower depths in the west and south of the array.

The above three studies show that two-dimensional magnetovariation arrays can be used to discover and map rock structures of high electrical conductivity. Figure 1.7 gives a schematic representation of their results (Gough, 1974).



Figure 1.7 Electrical conductivity of the upper mantle and tectonic provinces of western North America. The density of stippling gives a qualitative indication of variations of conductivity. NACP: North America Central Plains crustal conductor. From Gough (1974).

The density of stippling on the map gives a qualitative indication of variations of conductivity. The highest conductivities are found beneath the Wasatch Front mountains and southern Rockies: a ridge on the conductive mantle runs along the boundary between the Basin and Range province and the Colorado plateau below the Wasatch Front; under the Southern Rockies a conductive ridge runs from Wyoming to Mexico and the "Texas inland anomaly" lies on this ridge near the Mexican border. East of the front ranges of the Rocky Mountains the crust and mantle are in general resistive (except at the Central Plains crustal anomaly, which will be discussed later). The western region has highly conductive material in the upper mantle. The northern limit of the conductive mantle is close to the boundary between the Basin and Range province and the Columbia Plateaux, near  $43^{\circ}\text{N}$  (Camfield et al., 1971). However, below the Colorado plateau, probably the Middle Rockies and Wyoming Basin are more resistive.

An array of 41 three-component magnetometers was operated in 1972, between latitudes  $42^{\circ}$  and  $54^{\circ}\text{N}$  and longitudes  $98^{\circ}$  and  $109^{\circ}\text{W}$ . The objective was to study the North American Central Plains (NACP) conductive body. A long, narrow belt of rocks of very high electrical conductivity was mapped by the array, over a distance of 1800 km from southeastern Wyoming to the edge of the Canadian Shield in Saskatchewan (Alabi, Camfield and Gough, 1975).

During 1969, a two-dimensional array of 22 GR magnetometers was made over Oklahoma and North Texas and in the same year a second array of 28 GR magnetometers was operated between latitudes  $39^{\circ}$  and  $45^{\circ}\text{N}$  and longitudes  $89^{\circ}$  to  $100^{\circ}\text{W}$  (Porath and Dziewonski, 1971). Both arrays lie in the southern Great Plains province of the United States. Magnetovariation field anomalies were found to be associated with lateral changes in electrical conductivity of upper crust, sedimentary features, such as the deep Anadarko basin (thick conductive sediment, 38000 ft) and the Ouachita tectonic belt (boundary between the Paleozoic sediments in the west Texas and younger sediments in the Gulf Coast plains).

The very large EMSLAB magnetovariation array with 63 GR magnetometers was operated in 1985 in Oregon, Washington and adjoining areas, in conjunction with a seafloor array on the Juan de Fuca plate. The data are being analysed.

## 1.2 Results from Previous Magnetovariation and Magnetotelluric Work in Western Canada

### 1.2.1 Linear array results

Hyndman is the pioneer of the study of conductive structures in the western Canadian Cordillera. His linear array (A in Figure 1.5) ran at latitude  $49.5^{\circ}\text{N}$  from Victoria, British Columbia, eastward for 900 km to Lethbridge, Alberta (1963). He and his co-author found



(Hyndman, 1963; Cochrane and Hyndman, 1970) low Z values at all stations west of Crescent Valley (CRE), with large variations east of Kootenay Lake, British Columbia. The difference between two groups of stations was about a factor of 2.5. Lambert and Caner (1965) operated a linear array with four magnetometer stations in 1964 to investigate the "coast effect" in geomagnetic variations. The array ran from Tofino (TOF) on the west coast of Vancouver Island through Franklin River (FRA) and Westham (WES) to Abbotsford (ABB) on mainland. The results included records from the permanent Magnetic Observatory at Victoria (VIC). The enhancement in Z magnetovariation field were observed at the coastal station (Tofino) relative to the inland stations. Abbotsford and Westham overlapped Hyndman's array of 1963. Thus, Hyndman's array was extended to the west. Array B was made in 1965-1966 by Caner et al. (1967). It extended for about 500 km near latitude  $51^{\circ}\text{N}$  from Cache Creek (CAC), British Columbia, to Calgary (CAL), Alberta, and was supplemented by a short north-south linear array between CAC and Hope (HOP) in order to tie in with Hyndman's array.

Analyses of the records showed that the inland geomagnetic variation anomaly observed by Hyndman (1963) at Kootenay Lake, British Columbia, reappeared to the north between Revelstoke (REV) and Johnston Canyon (JOH). The change from low-Z (REV) to high-Z (JOH) occurred smoothly through a transition station (Golden), where there was an intermediate Z value. Array C ran north-south from Prince

George (PGE) to Cache Creek in an attempt to define a northern boundary of the low-Z region (Caner et al., 1971). The array tied in at Cache Creek with array B, and with the short north-south array between Cache Creek and Hope, which tied in with array A. There was no drastic discontinuity in Z magnetovariation fields between Cache Creek and Prince George. Previous work on the Cache Creek-Hope array indicated there was no discontinuity between Cache Creek and Victoria (Caner et al., 1967). This means that the northern extent of the discontinuity at Prince George, latitude  $54^{\circ}\text{N}$ , remains undefined.

Array D (Caner et al., 1971) included seven stations, in which two stations, Prince George and Quesnel (QUE), overlapped array C and the other five stations extended to Jasper (JAS) and Robb (ROB) in Alberta. the results were obtained that Robb and Jasper showed eastern-type high-Z; Barkerville, Quesnel, and Prince George were western-type low-Z; McBride and Valemount were low-Z at long periods, but transitional at the shorter periods (less than 30 min).

Array E with six stations was operated from Prince Rupert to Prince George (Dragert, 1970), with Prince George overlapping and providing tie-in with arrays C and D. Analyses of simultaneous recordings from all six stations showed no evidence for any first-order discontinuity between any of the stations; all the stations were clearly of the western low-Z type. The area in the dashed rectangle in Kootenay Lake region of British Columbia was covered by 20 stations

in a second-order survey.

An array of seven stations was operated during 1971 (Dragert and Clarke, 1977) from Clearwater, along Downie Creek, Rogers Pass and Nicholson, in British Columbia, into Banff, Cochrane and Suffield, in Alberta, between latitudes  $51.6^{\circ}$  and  $50.2^{\circ}$ N (shown in Figure 1.8). The results suggest the presence of three conductive structures: a near-surface conductor, which might be related to the sediments of the Rocky Mountain Trench, with depth extent is of 1-2 km and conductivity about 0.1 Siemens/m; a deeper conductive layer suggested by Caner (1970) and by Camfield and Gough (1975) to underlie the western Cordillera is probably at or dip to a depth of about 40-50 km beneath the Trench; and a third conductive structure which may be associated with a possible buried Precambrian rift in southwestern Alberta (Kanasewich, 1968).

The combined data from all linear arrays can be summarized as follows: (1) the main low-Z/high-Z discontinuity has been clearly identified. At or within a few km east of McBride ( $120.1^{\circ}$ W,  $53.3^{\circ}$ N) the discontinuity strikes NW-SE and continues at least to Valemount at about latitude  $52.5^{\circ}$ N and probably further to about  $51.5^{\circ}$ N. Somewhere between  $50.5^{\circ}$  and  $51.5^{\circ}$ N the discontinuity swings southward to an almost north-south strike, but this is not clearly defined by the data. (2) the low-Z area extends as far as the coast at latitude  $49^{\circ}$ - $50^{\circ}$ N and  $54^{\circ}$ - $55^{\circ}$ N and probably over the entire region contained within this range.

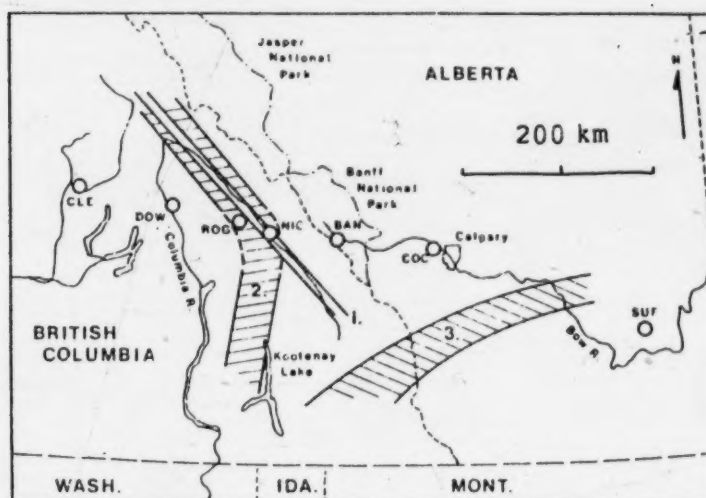


Figure 1.8 Station locations run by Dragert and Clarke in 1971 and the three deduced conductive structures. Modified from Dragert and Clarke (1977).

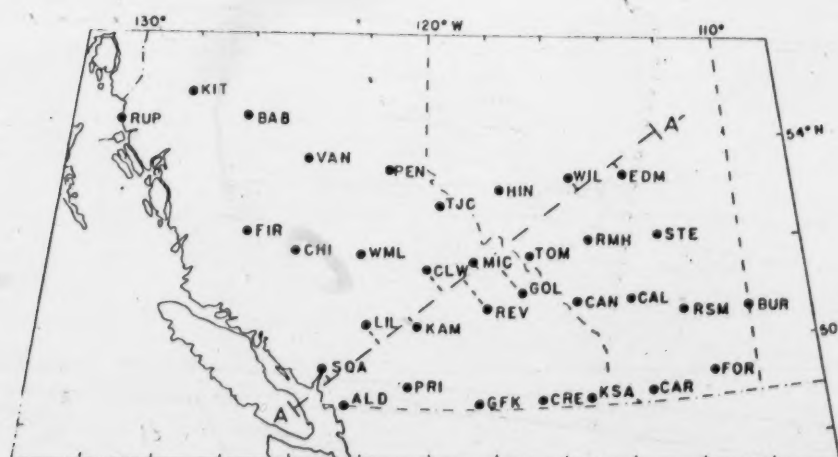


Figure 1.9 The location of the 1980A array of Gough et al. (1982).



(3) a conductive structure is probably associated with a buried Precambrian rift in southwestern Alberta and southeastern British Columbia. It strikes almost perpendicular to the edge of the low-Z.

### 1.2.2 Two-dimensional array results

Gough et al. (1982) operated a large, low-resolution magnetovariation array (1980A array), with 33 GR magnetometers, in June and July of 1980 in western Canada. The station locations are shown in Figure 1.9. It covered southwestern Canada south of the Edmonton-Prince Rupert highway, and was designed to discover anomalies of conductive structures for further study by means of arrays of higher resolution.

Analyses of magnetograms, FT anomaly maps, single-station transfer functions and artificial events showed the existence of three anomalies. (1) The regional low-Z anomaly was observed, with northeastern limit along the Rocky Mountain Trench in the northern half of the array, between  $52^{\circ}$ - $54^{\circ}$ N, as found in the earlier linear array work (Caner et al. 1971; Dragert 1973; Dragert and Clarke, 1977). Also a southward attenuation of Z magnetovariation fields across the plains, in southernmost Alberta near  $49.5^{\circ}$ N, was noted, as earlier observed by Camfield et al. (1971). (2) A local three-dimensional anomaly giving enhanced horizontal variation fields was found at Tete Jaune Cache in the Rocky Mountain Trench. (3) The southern Alberta-BC elongated

anomaly traverses southern Alberta and crosses the Rocky Mountains into the southeastern corner of British Columbia. The centre line of the anomaly runs northeast-southwest close to STE, CAL and CRE (Fig. 1.9).

In 1981 an array (1981A) with 33 GR magnetometers was centred at Tete Jaune Cache in the Rocky Mountain Trench west of Jasper and extended to the northeast across the Rocky Mountains to the Alberta Foothills and to the southwest across the Cariboo and Monashee Mountains (Bingham et al., 1985). The location of the stations is shown in Figure 1.10. Magnetograms and Fourier transform maps showed strong attenuation of the Z variation field southwest of the Rocky Mountain Trench, with very large Z amplitudes in the Main Ranges of the Rockies. The horizontal components showed an elongated anomaly along the Rocky Mountains Main Ranges and Trench, with three-dimensional features superimposed. The conductive structures include a highly conductive layer, probably in the lower crust, southwest of the trench and a conductive ridge rising into the upper crust near the edge of that layer (Fig. 1.10).

During 1980 September and October an array (1980B) of 31 GR magnetometers was operated over a known geothermal anomaly in the Rocky Mountains foothills near Edson and Hinton of Alberta (Ingham et al., 1983). The results of the investigation indicated the high conductivity associated with the geothermal anomaly.

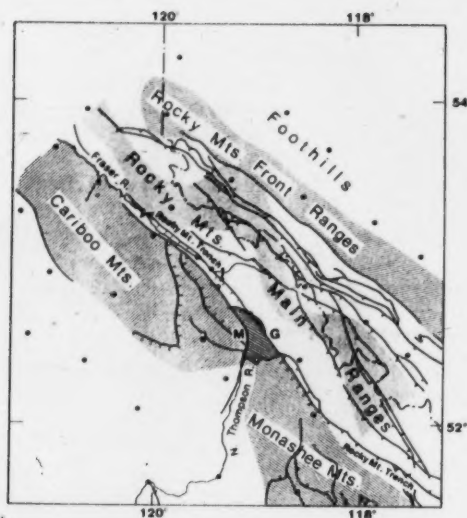


Figure 1.10 Stippling shows the location of the conductive ridge under the Main Ranges of the Rocky Mountains. Highly conductive rock occurs also as a layer, probably in the lower crust, everywhere southwest of the ridge. Solid points show locations of stations. MG is the Malton Gneiss block. From Bingham et al. (1985).



Figure 1.11 Magnetotelluric sounding stations in the plains of Alberta, up to 1967. From Reddy and Rankin (1972).

### 1.2.3 Magnetotelluric work in western Canada

Magnetotelluric sounding stations before 1967 in the plains of Alberta are shown in Figure 1.11 (Reddy and Rankin, 1972). Many of the magnetotelluric data obtained in western Canada were limited to periods less than 1000 seconds; so that the depth of exploration only reached the basement of the sedimentary basin.

Vozoff, Hasegawa and Ellis (1963) presented results of magnetotelluric measurements with periods  $< 400$  sec in the Edmonton area: Cooking lake, Onoway, Bon Accord, and Kavanagh. The results indicated an anisotropic layer with resistivity  $> 750$  ohm-meters at the base of the sediments (2.5 km). Rankin and Reddy (1969) reported an analysis and interpretation of magnetotelluric data, with periods 10-1000 seconds, obtained at the University of Alberta Geophysical Observatory, near Leduc, Alberta. The apparent resistivity curves reveal a strong anisotropy. From an anisotropic layered model, the anisotropy was found to lie in the Precambrian basement down to a depth of 14 km. The isotropic upper layer has a resistivity of 8 ohm-meters, which was characteristic of the sediment with a thickness of 2.5 km.

During the 1968 and 1969 field seasons, magnetotelluric measurements were made in the period range 1-1000 second at sixteen sites in central Alberta. The locations of the sites are shown Figure 1.12. Reddy and Rankin (Reddy, 1970; Reddy and Rankin, 1971; 1972) obtained tensor impedances and tensor apparent resistivities in the principal directions,



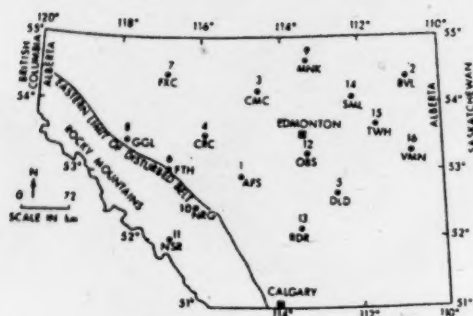


Figure 1.12 Magnetotelluric sounding station in central Alberta. 1. Alder Flats (AFS), 2. Bonnyville (BVL), 3. Camp Creek (CMC), 4. Carrot Creek (CRC), 5. Donalda (DLD), 6. Foot Hills (FTH), 7. Fox Creek (FXC), 8. Gregg Lake (GGL), 9. Meanook (MNK), 10. Nordegg (NRG), 11. NSR Crossing (NSR), 12. Observatoy (OBS), 13. Red Deer (RDR), 14. Smoky Lake (SML), 15. Two Hills (TWH), 16. Vermilion (VMN). Modified from Reddy and Rankin (1972).

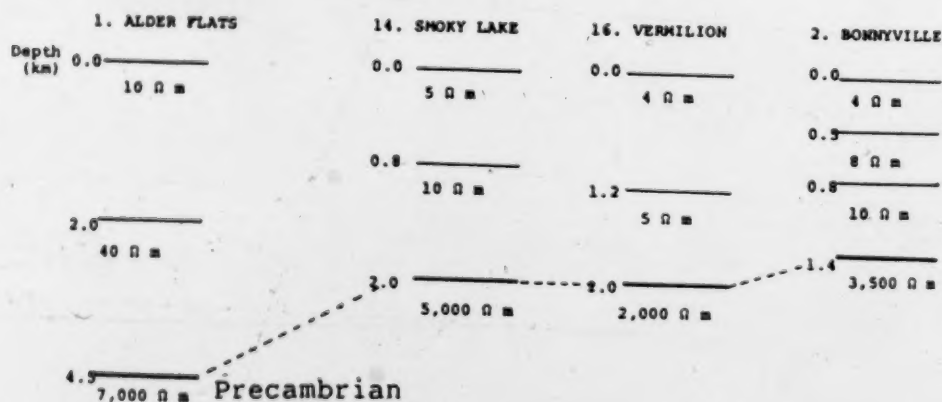


Figure 1.13 One-dimensional models for four magnetotelluric sounding sites in central Alberta. Modified from Reddy and Rankin (1971).

and the direction of principal axes were computed. The results indicated that all stations exhibited strong anisotropy of apparent resistivity for periods greater than 10 seconds. The direction of the major axis of tensor apparent resistivity was parallel to the strike of the Rocky Mountains and the sedimentary basin of western Canada.

The one-dimensional models derived from the apparent resistivities at four stations for E-polarization are presented in Figure 1.13 (Reddy and Rankin, 1971). The models yield a more highly resistive sedimentary section for Alder Flats and a low resistive sedimentary section for other three. Basement depths were read from well logs and used as invariant quantities in constructing models for each station. Peeples and Rankin (1973) presented magnetotelluric results from six stations in southern Alberta. The measurements were made in the period range 10 to 10<sup>3</sup> seconds. The stations are shown in Figure 1.14. The tensor apparent resistivities in the principal directions were computed and two dimensional numerical modelling techniques were applied. The results obtained indicated the presence of a buried geoelectric structure. Figure 1.15 showed the calculated  $\rho_{\text{PERP}}$  and  $\rho_1$  modes cross sections and model for the Precambrian rift. The direction of principal apparent resistivity ( $\rho_1$  axis) is 65° east of north. This principal direction is mainly controlled by the strike of the Rockies and the sedimentary basin of western Canada. The model displays an agreement with the structure found by Kanasewich



Figure 1.14 Locations of the six MT recording sites in southern Alberta. From Peeples and Rankin (1973).

et al. (Kanasewich, 1968 ; Kanasewich et al., 1969).

For study of the Earth's crust and upper mantle Niblett and Sayn-Wittgenstein (1960) made one of the earliest recordings of magnetotelluric fields in the western Canada. They used geoelectric and geomagnetic records at the Dominion Observatory research station at Meanook and analyzed continuous records of Ex, Ey, Hx and Hy for periods from about 40 seconds to 1000 seconds, and suggested a decrease from  $10^3$  to  $10^2$  ohm-meters in the resistivity between 60 and 100 km.

Srivastava et al. (1963) reported magnetotelluric results of six stations in the plains of Alberta. The locations of the stations were Meanook, Cooking Lake, Clive, Beiseker, Champion, and Cardston (see Figure 1.11). The data from Meanook for a period range of 9 to 3000 seconds, and from Beiseker for a period range of 9 to 1000 seconds, were analyzed in detail. The results obtained indicated that the depth of the top layer (sediment) with a resistivity 5-10 ohm-meters was 2-3 km, which corresponds well with the depth to Precambrian basement. A second layer was found to have a resistivity of 1100-1200 ohm-meters, with a discontinuity at a depth of 80-90 km to lower values. These estimates supported the results of Niblett and Sayn-Wittgenstein. Srivastava and Jacobs (1964) analysed records obtained during August 1961 from a field operation (Srivastava et al., 1963) and magnetograms and tellurograms recorded simultaneously at the Meanook observatory. Apparent



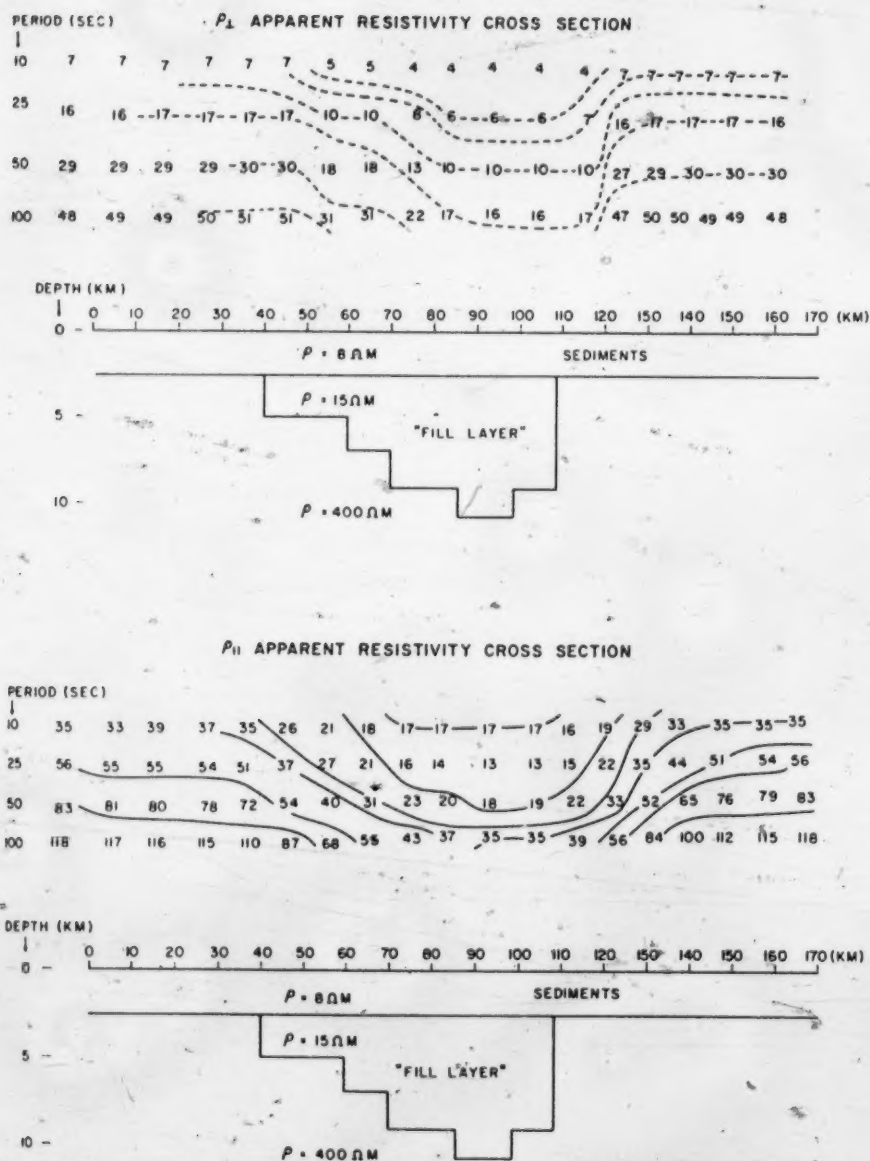


Figure 1.15 A graben model and calculated MT cross sections for TE and TM modes. From Peeples and Rankin (1973).

resistivity values for periods between 10 and 3000 seconds were obtained. Interpretation was done by matching the experimental curve to a set of master curves of layered Earth models.

A resistivity distribution obtained from one of the models giving the best fit to the observed data showed that the resistivity of the sediments and the depth to the Precambrian are 4.7 ohm-meters and 2.0 km, respectively; the second layer having resistivity 940 ohm-meters. A decrease to 94 ohm-meters in resistivity probably exists between 90 km and 140 km. The results agree quite well with the previous estimates of Niblett and Sayn-Wittgenstein, and Srivastava et al.. Vozoff and Ellis (1966) presented magnetotelluric results for three southern Alberta stations: Brooks, Beiseker and Vulcan. The thickness of sediment from three stations was about 2.35-3.70 km. Resistivity of the sediment section were calculated using logs from nearby wells. Their values vary between 0.5 and 200 ohm-meters. For Beiseker and Brooks a very resistive basement with a resistivity of  $10^4$  ohm-meters was observed and had a resistivity discontinuity, with a decrease to less than  $10^2$  ohm-meters, near 80-100 km. This is consistent with previous measurements (Srivastava et al. 1963). At Vulcan which is closest to the Rocky Mountains, a conductivity decrease from  $10^3$  to 30 ohm-meters occurred at a depth of about 35 km. During 1966 magnetotelluric data were obtained at Victoria, British Columbia, over a very wide range of periods. Caner

and Auld (1968) used these data for periods 2-15000 seconds to study conductive structure. The results are shown in Figure 1.16 and indicate the existence of a conductive layer of thickness  $10 \pm 3$  km and resistivity 100-125 ohm-meters at a depth of  $65 \pm 5$  km. A high resistivity zone with a resistivity of about 4000-5000 ohm-meters lies below this layer. There was no evidence for any further conducting zone down to a depth of 750 km

MT observations at a number of selected sites can be used to calibrate depths of conductive structures mapped from a magnetovariation study. Caner et al. (1969) designed a MT survey to provide such 'calibration' for their magnetovariation study in western Canada. Their stations were clustered in two groups: the cluster of three stations in the western region (Penticton, Grand Forks, Osoyoos) and the cluster (Pincher, Fernie) in the eastern region, shown in Figure 1.17. Figure 1.17 also shows the Vulcan station of Vozoff and Ellis (1966), and the Victoria sounding of Caner and Auld (1968). Data in a period range from 20 to 7500 seconds were analyzed. The combined results of five stations of Caner et al. and the Vulcan station of Vozoff and Ellis are shown Figure 1.18. A resistive upper crust with resistivity  $> 250$  ohm-meters and thickness 30-35 km was found under the eastern cluster, underlain by a conductive zone with resistivity 30-50 ohm-meters extending to a depth of at least 100 km. Under the western cluster the upper crustal layer is only  $15 \pm 5$  km thick and its resistivity is

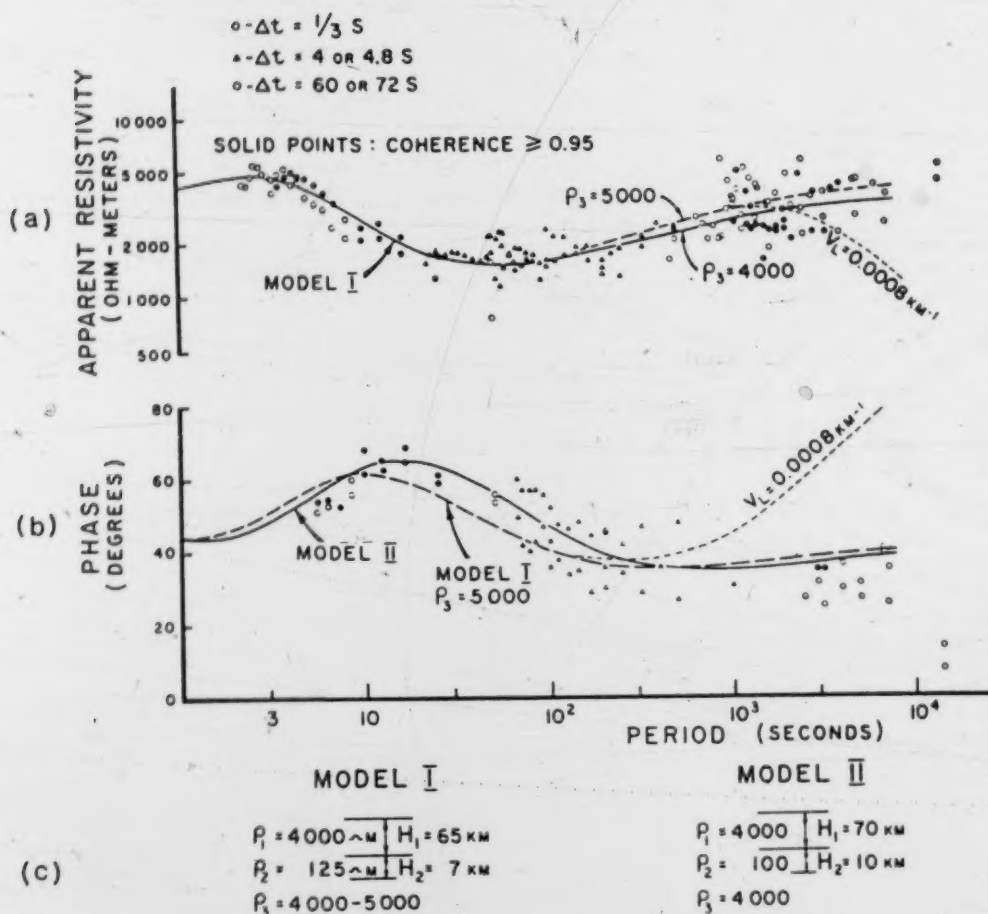


Figure 1.16 (a) Apparent resistivity (principal axis) plotted against period; (b) phase angle plotted against period; (c) conductivity structure models. From Caner and Auld (1968).



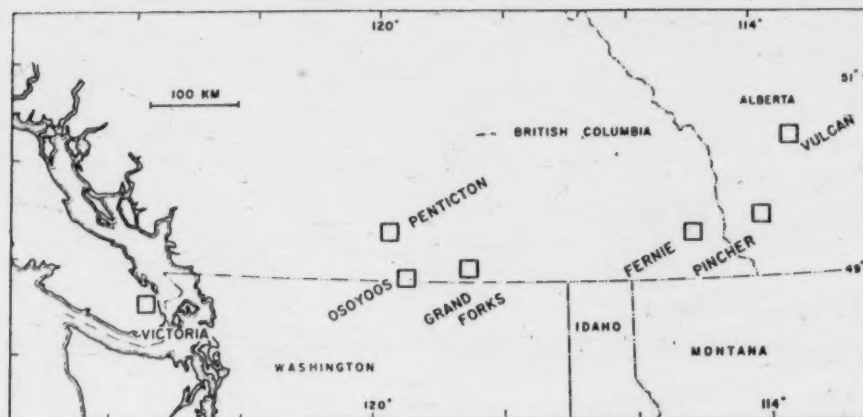


Figure 1.17 Location of magnetotelluric stations of Caner et al. (1969), as well as the Vulcan station of Vozoff and Ellis (1966) and the Victoria station of Caner and Auld (1968). From Caner et al. (1969).

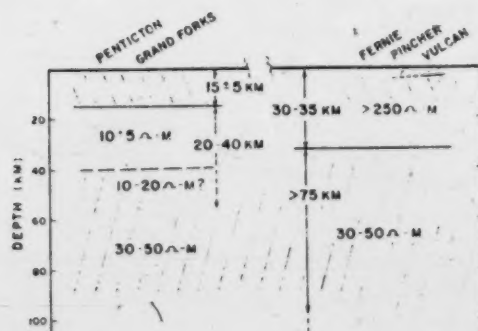


Figure 1.18 Conductive structure for southwestern Alberta and south central British Columbia, obtained from magnetotelluric sounding. Diagram from Caner et al. (1969).

indeterminate. It is underlain by a conductive zone with resistivity  $10 \pm 5$  ohm-meters and thickness 20-40 km. Below this layer the resistivity increases to a value of the same order (30-50 ohm-meters) as under that the eastern cluster. However it must be remembered that the interpretation of MT data suffers from some ambiguities, such as surface layer effects, source field effects and so on.

Following Whitham (1965) and Srivastava (1965) one can define

$$Q = \frac{Z}{H_x} = - \frac{\nu^2 P}{\partial P / \partial x} C(\omega, \nu, h, \rho) \quad (1.13)$$

(see (2.73), chapter 2). If one consider the same magnetovariation event recorded simultaneously at two stations, one can write

$$M = \frac{Q_1^2}{Q_2^2} = \frac{C_1^2}{C_2^2} \quad (1.14)$$

The term  $\nu^2 f(P)$  has been cancelled out. The ratio  $M$  can be evaluated as a function of  $\omega$  from observed field data. If the conductive structure at one station is reliably known, one can interpret unknown conductive structure at another station to fit the observed  $M$ , using various combinations of  $\nu$  and the conductive structure parameters ( $h, \rho$ ). Caner (1971) checked and improved their conductive structure model obtained from MT data (see Fig. 1.18), in terms of the above method and using geomagnetic depth sounding (GDS) data at Pincher and Penticton. This final conductivity structure model, based on combined MT and GDS data, is shown Figure

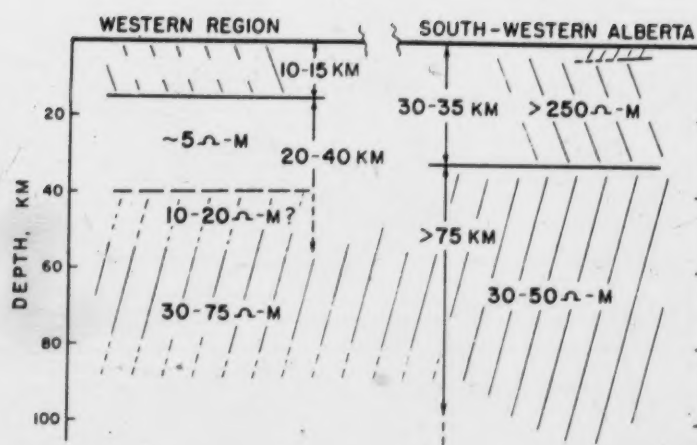


Figure 1.19 Final conductive structure model, based on combined MT and GDS data. From Caner (1971).

1.19. Pincher and Penticton were chosen to be close enough to obtain spatial coherence of the source fields and far enough to either side of the discontinuity to make it probable that the conductive structure under each approximate horizontal layers. The latter condition has been confirmed by the isotropy of the MT data at both stations. So at Penticton this final conductive model can be used as representative 'calibration' of the MVS for the low Z region in western Canada and at Pincher might be used as representative 'calibration' of the MVS for the high Z region in western Canada.

Dragert, Law and Sule(1980) carried out magnetotelluric soundings at three sites about 30 km apart in a line crossing the Lillooet Valley near Pemberton, British Columbia, to determine the regional conductivity structure and its relationship to the Meager Creek geothermal area. MT data in the period range 10 - 500 seconds were fitted by one-dimensional models with a good ( $< 100$  ohm-m) conductive zone beneath Pemberton, beneath the highly resistive ( $> 10000$  ohm-m) crystalline complex (15-25 km). The cause of this enhanced conductivity was believed to be hydration or partial melting as suggested for other areas of the Canadian Cordillera. This conductive layer is similar to that suggested by Caner (1971) for the Cordillera in western Canada.



### 1.3 Previous Knowledge about Conductive Structures of the Canadian Cordillera

Magnetovariation studies with linear and two-dimensional arrays have been used to discover and map the conductive structures in western Canada and magnetotelluric soundings have been used to locate depth and conductivity value of the conductive layer. Gough (1986a) summarized the results of magnetovariation and magnetotelluric studies and gave a schematic representation of the conductive structures in western Canada, as shown in Figure 1.20, together with the boundaries of the five tectonic provinces. The three major conductive structures are included: (1) Canadian Cordilleran Regional (CCR) conductor: The northeastern limit of the CCR conductor lies along the Rocky Mountain Trench and Main Ranges north of  $51^{\circ}\text{N}$ , and then swings southward to almost north-south along the Kootenay Arc between  $49^{\circ}\text{N}$  and  $51^{\circ}\text{N}$ . Its western edge is poorly located and its northern limit has not been determined. The depth to the upper surface of CCR conductor may be 15-30 km, and its thickness between 30 and 70 km, with a conductivity near 0.1 S/m. These estimates are, however, based on the small number of MT soundings already noted. This conductor appears to start in the lower crust and extend downward into the upper mantle. The CCR conductor causes the pronounced regional attenuation in the Z magnetovariation fields. (2) The Northern Rockies (NR) conductor lies beneath the Rocky Mountain Main Ranges along

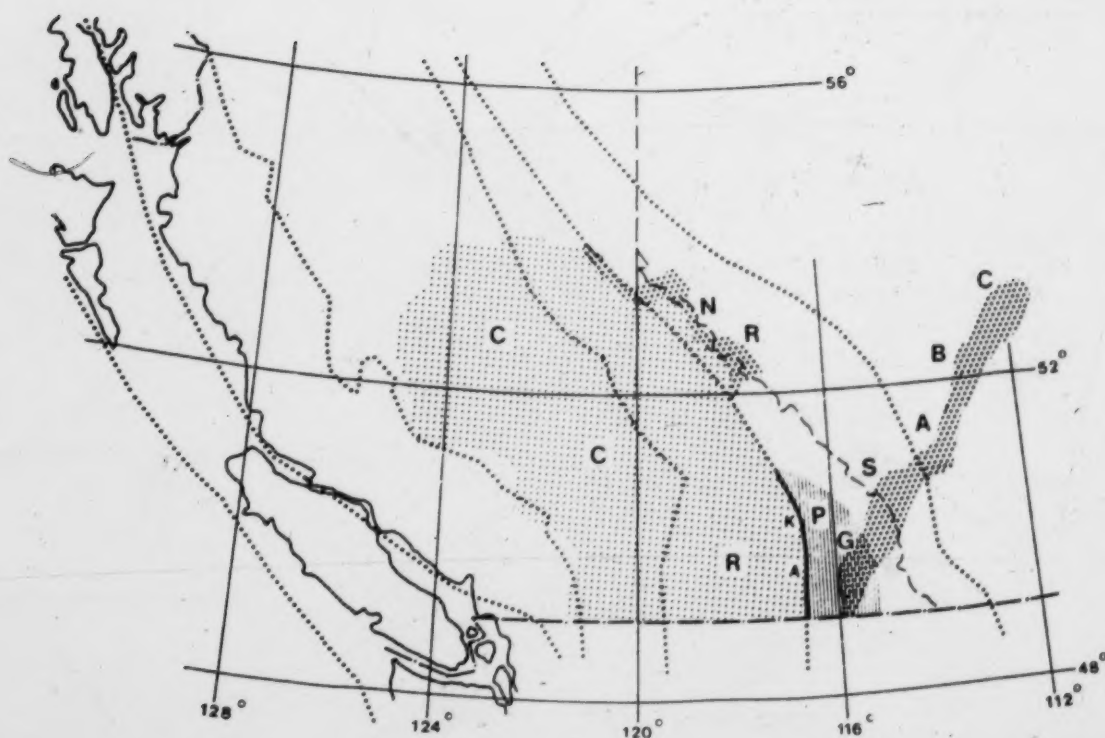


Figure 1.20 Schematic representation of three conductive structures in western Canada: CCR, Canadian Cordilleran Regional conductor; NR North Rockies conductor; SABC, Southern Alberta-British Columbia conductor; PG, Purcell Geanticline; KA, Kootenay Arc. From Gough (1986a).

their strike, between  $54^{\circ}\text{N}$  and  $52^{\circ}\text{N}$ . It can be regarded as a conductive ridge formed near the northeastern edge of the CCR conductor. It rises to about a depth of 10 km in the upper crust and has conductivity of the order of 1 S/m. This NR conductor produces large anomalies in the magnetovariation fields. (3) Southern Alberta-British Columbia (SABC) conductor: This is an elongated conductive body in the lower crust or upper mantle. It strikes northeast-southwest across southern Alberta and the southeastern corner of British Columbia.

## 2. PRINCIPLES OF ELECTROMAGNETIC INDUCTION IN THE EARTH

### 2.1 Basic theory

The theory for magnetovariation studies is based on the Maxwell equations. In SI units these are:

$$\nabla \times \vec{E} = -\partial \vec{B} / \partial t \quad (2.1)$$

$$\nabla \times \vec{H} = \vec{J} + \partial \vec{D} / \partial t \quad (2.2)$$

$$\nabla \cdot \vec{D} = \rho \quad (2.3)$$

$$\nabla \cdot \vec{B} = 0 \quad (2.4)$$

where

$\vec{E}$  = the electric field

$\vec{H}$  = the magnetic field

$\vec{D}$  = the electric flux density or displacement

$\vec{B}$  = the magnetic flux density or magnetic induction

$\vec{J}$  = the electric current density

$\rho$  = the electric charge density

In a linear and isotropic medium one has simple constitutive relationships

$$\vec{D} = \epsilon \vec{E} \quad (2.5)$$

$$\vec{B} = \mu \vec{H} \quad (2.6)$$

$$\vec{J} = \sigma \vec{E} \quad (2.7)$$

where  $\epsilon$  is the dielectric constant of the medium,  $\mu$  its magnetic permeability and  $\sigma$  its conductivity.

For fields of period  $T$ ,  $\partial \vec{D} / \partial t$  is of order  $\epsilon \vec{E} / T$ . Hence,  $\partial \vec{D} / \partial t \ll \vec{J}$  if  $T \gg \epsilon / \sigma \approx 1 / \mu \sigma C^2$ . In SI units,  $C = 3 \times 10^8$  m/s and  $\mu = \mu_0 = 4\pi \times 10^{-7}$  Henry/m, so that, even if  $\sigma$  is as small as  $10^{-7}$  S/m,  $T$  has only to be much greater than one second.



Therefore, in the magnetovariation study we always neglect the displacement current  $\partial \vec{D}/\partial t$ , compared to the conduction current  $\vec{J}$ , and (2.2) reduces to

$$\nabla \times \vec{H} = \vec{J} \quad (2.8)$$

Taking the divergence of (2.8) and using (2.7), we have

$$\nabla \cdot \vec{E} = -\frac{\nabla \sigma}{\sigma} \cdot \vec{E} \quad (2.9)$$

Taking the curl of (2.1) and using (2.8), (2.6) and (2.7), one can obtain

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\sigma \mu \partial \vec{E} / \partial t + \frac{\nabla \mu}{\mu} \times (\nabla \times \vec{E}) \quad (2.10)$$

Similarly,

$$\nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = -\sigma \mu \partial \vec{H} / \partial t + \frac{\nabla \sigma}{\sigma} \times (\nabla \times \vec{H}) \quad (2.11)$$

(2.10) and (2.11) are satisfied by  $\vec{E}$  and  $\vec{H}$  inside any isotropic conductor. When the conducting medium is uniform, equations (2.9), (2.10) and (2.11) reduce to

$$\nabla \cdot \vec{E} = 0 \quad (2.12)$$

$$\nabla^2 \vec{E} = \sigma \mu \partial \vec{E} / \partial t \quad (2.13)$$

$$\text{and } \nabla^2 \vec{H} = \sigma \mu \partial \vec{H} / \partial t \quad (2.14)$$

(2.13) and (2.14) are diffusion equations for  $\vec{E}$  and  $\vec{H}$ . Thus low frequency EM fields in an uniform conductor are propagated according to diffusion equations, not wave equations.

Linearity is a fundamental feature of the electromagnetic field in the Earth (Rokityansky, 1982, p.28). This enables one to apply the superposition principle

to spatial or time harmonics of the field. For time varying fields of the form

$$\vec{E} = \vec{E} e^{i\omega t} \quad (2.15)$$

$$\vec{H} = \vec{H} e^{i\omega t} \quad (2.16)$$

equations (2.13) and (2.14) can be written

$$\nabla^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} = i\omega\sigma\mu \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} \quad (2.17)$$

Outside the conductor, where  $\sigma = 0$ , (2.17) reduces to Laplace's equations:

$$\nabla^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} = 0 \quad (2.18)$$

## 2.2 Matrix representation of electromagnetic fields in the Earth

When the medium is uniform, equations (2.1) and (2.2) can be written as , with use of (2.12) and (2.4),

$$\nabla^2 \vec{E} = \sigma\mu \partial \vec{E} / \partial t + \epsilon\mu \partial^2 \vec{E} / \partial^2 t \quad (2.2.1)$$

$$\text{and} \quad \nabla^2 \vec{H} = \sigma\mu \partial \vec{H} / \partial t + \epsilon\mu \partial^2 \vec{H} / \partial^2 t \quad (2.2.2)$$

$$\text{or} \quad \nabla^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} = k^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} \quad (2.2.3)$$

$$\text{where} \quad k^2 = i\sigma\mu\omega - \epsilon\mu\omega^2 \quad (2.2.4)$$

The Fourier transform of  $\vec{E}$  and  $\vec{H}$  are,

$$\vec{e}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{E}(x,y,z,t) \exp[i(k_x x + k_y y - \omega t)] dx dy dt \quad (2.2.5)$$

$$\vec{h}(z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{h}(x, y, z, t) \exp[i(k_x x + k_y y - \omega t)] dx dy dt \quad (2.2.6)$$

Taking Fourier transform of (2.2.3), one obtains

$$\vec{e}'' - \eta^2 \vec{e} = 0 \quad (2.2.7)$$

$$\text{and } \vec{h}'' - \eta^2 \vec{h} = 0 \quad (2.2.8)$$

$$\text{where } \eta = \sqrt{(k_x^2 + k_y^2 + k^2)}, \quad \text{Re } \eta > 0 \quad (2.2.9)$$

and the prime denotes differentiation over  $z$ .

The solutions of equations (2.2.7) and (2.2.8) are,

$$\vec{e} = \vec{e}^+ \exp[+\eta z] + \vec{e}^- \exp[-\eta z] \quad (2.2.10)$$

$$\text{and } \vec{h} = \vec{h}^+ \exp[+\eta z] + \vec{h}^- \exp[-\eta z] \quad (2.2.11)$$

where  $\vec{e}^+$  and  $\vec{e}^-$ ,  $\vec{h}^+$  and  $\vec{h}^-$  are constant vectors specified from the boundary conditions.

The tangential components of  $\vec{e}$  and  $\vec{h}$  can be written as a form of the matrix

$$\begin{bmatrix} e_x \\ e_y \\ h_x \\ h_y \end{bmatrix} = \begin{bmatrix} e_x^+ & e_x^- & 0 & 0 \\ e_y^+ & e_y^- & 0 & 0 \\ 0 & 0 & h_x^+ & h_x^- \\ 0 & 0 & h_y^+ & h_y^- \end{bmatrix} \begin{bmatrix} e^{+\eta z} \\ e^{-\eta z} \\ e^{+\eta z} \\ e^{-\eta z} \end{bmatrix} \quad (2.2.12)$$

$$\text{or } A = BC \quad (2.2.13)$$

$$\text{where } A = [e_x, e_y, h_x, h_y]^T \quad (2.2.14)$$

$$B = \begin{bmatrix} e_x^+ & e_x^- & 0 & 0 \\ e_y^+ & e_y^- & 0 & 0 \\ 0 & 0 & h_x^+ & h_x^- \\ 0 & 0 & h_y^+ & h_y^- \end{bmatrix} \quad (2.2.15)$$

$$\text{and } C = [e^{+\eta z}, e^{-\eta z}, e^{+\eta z}, e^{-\eta z}]^T \quad (2.2.16)$$

If the Earth consists of  $n$  plane homogenous layers of electrical conductivity  $\sigma_n$  and thickness  $d_n$ , the vector  $A$  at interfaces should be continuous. For example, at the interface between the  $n$ th and the  $(n+1)$ th layers, we have

$$A_n = A_{n+1} \quad (2.2.17)$$

This method can be extended in a fashion like that used in seismology (Aki and Richards, 1980, p.163 and p.259)

### 2.3 "Flat Earth" Models

We start by treating the Earth as a semi-infinite conductor occupying the half-space  $z > 0$  of Cartesian coordinates. The conductivity  $\sigma$  is assumed to be a function of  $z$  only. Within the Earth, the conductivity  $\sigma(z)$  will probably vary with depth to a much greater extent than either  $\epsilon$  or  $\mu$ . So we take  $\epsilon$  and  $\mu$  as constants. We assume that a time-varying magnetic field, with arbitrary distribution and having its source in the region  $-h < z < 0$ , induces currents in the conductor. These induced currents, in turn, produce secondary, or induced, magnetic field. We wish to determine the total magnetic field at the surface  $z = 0$ , which consists of the time-varying source and secondary fields.

Because  $\nabla \mu = 0$ , equation (2.10) for  $\vec{E}$  reduces to

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\sigma \mu \partial \vec{E} / \partial t \quad (2.19)$$



As  $\sigma$  depends on  $z$  only, using Cartesian coordinates ( $x, y, z$ ) we can write equation (2.9) as

$$\sigma(z) \left[ \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] + E_z \frac{\partial \sigma}{\partial z} = 0 \quad (2.20)$$

It can be shown that, with the approximations usual in electromagnetic induction in the Earth, the currents induced by a varying external magnetic field flow parallel to the surface of the Earth at all depths. This is because all layers have conductivity  $\gg$  the space above the surface (Lahiri and Price, 1939; Price, 1950). Hence

$$i_z = \sigma E_z = 0 \quad (2.21)$$

and equation (2.20) becomes

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} = 0 \quad (2.22)$$

and from (2.17) we again obtain the diffusion equation

$$\nabla^2 \vec{E} = \sigma \mu \partial \vec{E} / \partial t \quad (2.11)$$

Assuming that elementary solutions of (2.11) have the form (Price, 1962; 1967)

$$\vec{E} = e^{i\omega t} Z(z) \vec{F}(x, y) \quad (2.23)$$

By (2.21) and (2.22),  $\vec{F}$  can be written as

$$\vec{F}(x, y) = \left[ \frac{\partial P}{\partial y}, -\frac{\partial P}{\partial x}, 0 \right] \quad (2.24)$$

where  $P$  is a function of  $x$  and  $y$  only. On substituting (2.23) and (2.24) in (2.11), we obtain

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \vec{F} = \left[ -\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + i\omega\sigma\mu \right] \vec{F} \quad (2.25)$$

Now the coefficient of  $\vec{F}$  on the righthand side of (2.25) can be written as a real constant  $-\nu^2$ , since it is independent of  $x$  and  $y$  and the left-hand side is independent of  $z$ .

Equation (2.25) then leads to

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \nu^2 \right] \vec{F} = 0 \quad (2.26)$$

$$\frac{\partial^2 Z}{\partial z^2} = [\nu^2 + i\omega\sigma\mu] Z \quad (2.27)$$

Substituting (2.24) in (2.26), we obtain

$$\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \nu^2 \right] P = 0 \quad (2.28)$$

Combining (2.23) and (2.24) we have

$$\vec{E} = e^{i\omega t} Z(z) \left[ \frac{\partial P}{\partial y}, -\frac{\partial P}{\partial x}, 0 \right] \quad (2.29)$$

From equation (2.1), we find that the corresponding magnetic field is

$$\vec{H} = \frac{i}{\omega\mu} \left[ \frac{\partial Z}{\partial z} \frac{\partial P}{\partial x}, \frac{\partial Z}{\partial z} \frac{\partial P}{\partial y}, \nu^2 Z P \right] e^{i\omega t} \quad (2.30)$$

Outside the conductor, where  $\sigma = 0$ , equation (2.27) has the solution

$$Z_{out}(z) = Ae^{-\nu z} + Be^{\nu z} \quad (2.31)$$

Substituting the value of  $Z_{out}(z)$  from (2.31) in (2.30) we get

$$\vec{H}_{out} = \frac{i\nu}{\omega\mu_0} \nabla \left[ (-Ae^{-\nu z} + Be^{\nu z}) P(x, y) \right] e^{i\omega t} \quad (2.32)$$

Outside the conductor, from (2.8) we have

$$\nabla \times \vec{H}_{out} = \vec{J}_{out} = 0 \quad (2.33)$$

$$\vec{H}_{out} = -\nabla \Omega_{out} \quad (2.34)$$

Hence the scalar potential of the magnetic field outside the conductor ( $z < 0$ ) is

$$\Omega_{out} = \frac{i\nu}{\omega\mu_0} \left[ (Ae^{-\nu z} - Be^{\nu z}) P(x, y) \right] e^{i\omega t}$$

$$= \left[ (A_p e^{-\nu z} + A_s e^{\nu z}) P(x, y) \right] e^{i\omega t} \quad (2.35)$$

where

$$A_p = \frac{i\nu A}{\omega\mu_0} \quad (2.36)$$

$$A_s = -\frac{i\nu}{\omega\mu_0} B \quad (2.37)$$

If  $\nu$  is taken as real and positive, the term involving  $e^{-\nu z}$  in (2.35) corresponds to a source field, or so called primary field, in the region  $-h < z < 0$ , and therefore represents an inducing field, whereas the term involving  $e^{\nu z}$  represents the field of the induced currents, or so called secondary field.

### 2.3.1 Electromagnetic induction in a uniform half-space

Inside the conductor if we let  $\sigma$  be a constant, equation (2.27) has solution

$$Z_{in}(z) = A'e^{-\theta z} + B'e^{\theta z} \quad (2.38)$$

$$\text{Where } \theta^2 = \nu^2 + i\omega\mu\sigma \quad (2.39)$$

In the case of induction by an external magnetic field,  $Z_{in}$  in the conductor should approach zero as  $z$  tends to infinity, so that  $B' = 0$ . Hence

$$Z_{in} = A'e^{-\theta z} \quad (2.40)$$

Since the tangential components of  $\vec{E}$  are continuous at  $z = 0$ , the same function  $P(x, y)$  must appear in the expression for  $\vec{E}$  either outside or inside the conductor. Hence we obtain

$$\vec{E}_{out} = Z_{out} \left[ \frac{\partial P}{\partial y}, -\frac{\partial P}{\partial x}, 0 \right] e^{i\omega t} \quad (2.41)$$

$$\vec{H}_{out} = \frac{i}{\omega\mu_0} \left[ \frac{\partial Z_{out}}{\partial z} \frac{\partial P}{\partial x}, \frac{\partial Z_{out}}{\partial z} \frac{\partial P}{\partial y}, \nu^2 Z_{out} P \right] e^{i\omega t} \quad (2.42)$$

and

$$\vec{E}_{in} = Z_{in} \left[ \frac{\partial P}{\partial y}, -\frac{\partial P}{\partial x}, 0 \right] e^{i\omega t} \quad (2.43)$$

$$\vec{H}_{in} = \frac{i}{\omega\mu} \left[ \frac{\partial Z_{in}}{\partial z} \frac{\partial P}{\partial x}, \frac{\partial Z_{in}}{\partial z} \frac{\partial P}{\partial y}, \nu^2 Z_{in} P \right] e^{i\omega t} \quad (2.44)$$

Applying the boundary conditions on  $\vec{B}$  and  $\vec{H}$ , from (2.42) and (2.44) we obtain

$$A + B = A' \quad (2.45)$$

$$\frac{\nu}{\mu} (A - B) = \frac{\nu}{\mu} A' \quad (2.46)$$

or

$$B = \frac{\nu\mu_r - \theta}{\nu\mu_r + \theta} A \quad (2.47)$$

$$A' = \frac{2\nu\mu_r}{\nu\mu_r + \theta} A \quad (2.48)$$

where  $\mu_r (= \frac{\mu}{\mu_0})$  is the relative magnetic permeability of the conducting medium.

Substituting (2.31) in (2.42) and using relations (2.36) and (2.37) we can obtain the total magnetic field  $\vec{H}_t$  at  $z = 0$ ,

$$\vec{H}_t = - \left[ (A_p + A_s) \frac{\partial P}{\partial x}, (A_p + A_s) \frac{\partial P}{\partial y}, \nu (A_p - A_s) P \right] e^{i\omega t} \quad (2.49)$$

with

$$\vec{H}_p = - \left[ A_p \frac{\partial P}{\partial x}, A_p \frac{\partial P}{\partial y}, \nu A_p P \right] e^{i\omega t} \quad (2.50)$$

represents inducing field at  $z = 0$  and

$$\vec{H}_s = - \left[ A_s \frac{\partial P}{\partial x}, A_s \frac{\partial P}{\partial y}, -\nu A_s P \right] e^{i\omega t} \quad (2.51)$$



represents induced field at  $z = 0$ .

Using equation (2.6), the magnetic induction measured by a magnetometer can be express as

$$\vec{B}_t = -\mu_0 \left[ (A_p + A_s) \frac{\partial P}{\partial x}, (A_p + A_s) \frac{\partial P}{\partial y}, \nu (A_p - A_s) P \right] e^{i\omega t} \quad (2.49a)$$

with

$$\vec{B}_p = -\mu_0 \left[ A_p \frac{\partial P}{\partial x}, A_p \frac{\partial P}{\partial y}, \nu A_p P \right] e^{i\omega t} \quad (2.50a)$$

and

$$\vec{B}_s = -\mu_0 \left[ A_s \frac{\partial P}{\partial x}, A_s \frac{\partial P}{\partial y}, -\nu A_s P \right] e^{i\omega t} \quad (2.51a)$$

where  $\mu_0 = 4\pi \times 10^{-7}$  Henry/meter.

From (2.49)-(2.51) and (2.49a)-(2.51a) it is clear that the effect of the induced field is to increase the horizontal components of the source field and decrease the vertical component.

Eliminating A and B from (2.36), (2.37) and (2.47), we get

$$A_s = \frac{\theta - \nu \mu_r}{\theta + \nu \mu_r} A_p \quad (2.52)$$

When  $\mu_0 \omega$  is very large,  $\theta \gg \nu$ , from (2.52)  $A_s \approx A_p$  (as  $\mu_r \approx 1$ ). In the limiting case  $A_s = A_p$ , the total magnetic induction at  $z = 0$  becomes

$$\vec{B}_t = -\mu_0 \left[ 2A_p \frac{\partial P}{\partial x}, 2A_p \frac{\partial P}{\partial y}, 0 \right] e^{i\omega t} \quad (2.53)$$

That is, in the limit of a uniform, highly conducting half

space and a high-frequency source field, the total magnetic induction on the surface of the half space has zero vertical component and has horizontal components of twice the intensity of those of the source field.

For a uniform half-space with  $\sigma = \text{constant}$ , suppose an external potential of the form (see (2.35))

$$\Omega_e = A_P P(x, y) e^{(-\nu z + i\omega t)} \quad (2.54)$$

then the magnetovariation field (magnetic induction) at a depth  $z$  can be obtained by combining (2.44), (2.40), (2.48), (2.36) and (2.6),

$$\vec{B}_{in} = \frac{2\nu\mu\mu_0}{\nu\mu + \theta\mu_0} \left[ -\theta \frac{\partial P}{\partial x}, -\theta \frac{\partial P}{\partial y}, \nu^2 P \right] e^{(-\theta z + i\omega t)} \quad (2.55)$$

where

$$\theta = \frac{1}{\sqrt{2}} \{ [(\gamma^2 + \nu^2)^{1/2} + \nu^2]^{1/2} + i [(\gamma^2 + \nu^2)^{1/2} - \nu^2]^{1/2} \} \quad (2.56)$$

$$\text{and } \gamma^2 = \omega\mu\sigma \quad (2.57)$$

The real part of  $\theta$  represents the attenuation of the magnetovariation field with depth. Thus the attenuation factor for the magnetovariation field is obtained from (2.55)

$$Q_z = \exp\left\{-\frac{z}{\sqrt{2}}[(\gamma^2 + \nu^2)^{1/2} + \nu^2]^{1/2}\right\} \quad (2.58)$$

The skin depth at which  $Q_z = 1/e$ , is

$$D_s = \sqrt{2} / [(\gamma^2 + \nu^2)^{1/2} + \nu^2]^{1/2} \quad (2.59)$$

The reciprocal of the parameter  $\nu$  describes the horizontal scale of the source field ( $\lambda = 2\pi/\nu$ , where  $\lambda$  represents the

wavelength of the field). If  $\nu = 0$ , that is, when the source field is uniform, then (2.59) reduces to the ordinary skin depth formula in SI units,

$$D_s = \sqrt{2/\omega\mu\sigma} \quad (2.60)$$

From (2.59) and (2.60) we can see that as  $\sigma$  or  $\omega$ , or both increases,  $D_s$  decreases. If  $\nu$  is not too small compared with  $\sqrt{(\omega\mu\sigma)}$  (see (2.57) and (2.59)), the skin depth is significantly affected by the horizontal scale of the source field: the skin depth increases as the horizontal scale of the source field increases.

### 2.3.2 Electromagnetic induction in a horizontally layered half-space

An alternative method to the matrix method is presented here.

If the half-space is composed of  $N$  layers (the lowest one extending to infinity), the  $n$ th layer having electrical conductivity  $\sigma_n$  and thickness  $h_n$ . Suppose  $\sigma_n$  remains constant, so (2.38) is valid for the  $n$ th layer. Substituting (2.38) in (2.43) and (2.44) and taking the ratio in the  $n$ th layer at depth  $z = z_1$  from the surface, we get

$$\left. \frac{E_x}{H_y} \right|_{z=z_1} = - \frac{i\omega\mu (B'e^{\theta_n z_1} + A'e^{-\theta_n z_1})}{\theta_n (B'e^{\theta_n z_1} - A'e^{-\theta_n z_1})} \quad (2.61)$$

$$\text{where} \quad \theta_n^2 = \nu^2 + i\omega\mu\sigma_n \quad (2.62)$$

(We have omitted subscripts "in" in  $\vec{E}$  and  $\vec{H}$ . (2.61) can be written as

$$Z_1 = \left[ \frac{E_x}{H_y} \right]_{z=z_1} = - \frac{i\omega\mu}{\theta_n} \coth \left[ \theta_n z_1 + \ln \left( \frac{B'}{A'} \right)^{1/2} \right] \quad (2.63)$$

where  $Z_1$  is called the impedance at  $z=z_1$ . using (2.63) we can find the impedance  $Z_2$  at a depth  $z=z_2$  in the  $n$ th layer, where  $z_2 < z_1$ , as

$$Z_2 = \left[ \frac{E_x}{H_y} \right]_{z=z_2} = - \frac{i\omega\mu}{\theta_n} \coth \left[ \theta_n z_2 + \ln \left( \frac{B'}{A'} \right)^{1/2} \right] \quad (2.64)$$

From 2.63) we have

$$\ln \left( \frac{B'}{A'} \right)^{1/2} = - \coth^{-1} \left[ \frac{Z_1 \theta_n}{i\omega\mu} \right] - \theta_n z_1 \quad (2.65)$$

Substituting value of  $\ln \left( \frac{B'}{A'} \right)^{1/2}$  of (2.65) into (2.64) we get

$$Z_2 = - \frac{i\omega\mu}{\theta_n} \coth \left[ \theta_n (z_2 - z_1) - \coth^{-1} \left[ \frac{Z_1 \theta_1}{i\omega\mu} \right] \right] \quad (2.66)$$

If we take  $z_2 = 0$  and  $z_1 = h_1$ , then we get the surface impedance

$$Z(0) = \lim_{\substack{z_2 \rightarrow 0 \\ z_1 \rightarrow h_1}} Z_2 = - \frac{i\omega\mu}{\theta_1} \coth \left[ - \theta_1 h_1 - \coth^{-1} \left( \frac{Z_1 \theta_1}{i\omega\mu} \right) \right] \quad (2.67)$$

Due to the continuity of the impedance ( $E_x/H_y$ ) at the interface, we can express  $Z_1$  in terms of  $Z_2$ ,  $Z_2$  in terms of  $Z_3$  and so on, using (2.67) and noting

$$\begin{aligned} Z_{N-1} &= - \frac{i\omega\mu}{\theta_N} \coth \left[ - \theta_N h_N - \coth^{-1} \left( \frac{Z_N \theta_N}{i\omega\mu} \right) \right] \\ &= \frac{i\omega\mu}{\theta_N}, \text{ as } h_N \rightarrow \infty \end{aligned} \quad (2.68)$$

Hence we obtain

$$Z(0) = \left[ \frac{E_x}{H_y} \right]_{z=0} = \frac{i\omega\mu}{\theta_1} \coth \left[ \theta_1 h_1 + \coth^{-1} \left[ \frac{\theta_1}{\theta_2} \coth \left[ \theta_2 h_2 + \right. \right. \right]$$

$$\coth^{-1}\left(\frac{\theta_2}{\theta_3}\coth(\theta_3h_3 + \dots + \coth^{-1}\left(\frac{\theta_{N-1}}{\theta_N}\right))\right)\right] \quad (2.69)$$

If we set

$$C = \frac{1}{\theta_1}\coth\left[\theta_1h_1 + \coth^{-1}\left[\frac{\theta_1}{\theta_2}\coth\left[\theta_2h_2 + \coth^{-1}\left(\frac{\theta_2}{\theta_3}\coth(\theta_3h_3 + \dots + \coth^{-1}\left(\frac{\theta_{N-1}}{\theta_N}\right))\right)\right]\right]\right] \quad (2.70)$$

then we get

$$Z(0) = i\omega\mu C \quad (2.71)$$

$$\text{or} \quad C = \frac{Z(0)}{i\omega\mu} \quad (2.72)$$

where C is known as the inductive response function (Schmucker, 1970; Lilley, 1975, Jones, 1980).

Similarly, we obtain

$$\frac{H_z}{H_x} = - \frac{\nu^2 PC}{\partial P / \partial x} \quad (2.73)$$

$$\text{or} \quad C = - \frac{H_z}{H_x} \frac{\partial P / \partial x}{\nu^2 P} = - \frac{B_z}{B_x} \frac{\partial P / \partial x}{\nu^2 P} \quad (2.74)$$

From Schmucker (1970) we have two other relationships

$$C = \frac{H_z}{\partial H_x / \partial x + \partial H_y / \partial y} = \frac{B_z}{\partial B_x / \partial x + \partial B_y / \partial y} \quad (2.75)$$

$$\text{and} \quad \frac{\partial H_x / \partial x + \partial H_y / \partial y}{H_x + H_y} = \frac{\partial B_x / \partial x + \partial B_y / \partial y}{B_x + B_y} = - \frac{\nu^2 P}{\partial P / \partial x + \partial P / \partial y} \quad (2.76)$$

When the inductive response function is determined at a series of frequencies from observed electromagnetic data, by using (2.72) or (2.74) or (2.75), 1-D modeling by means of (2.70) (Porath, Gough and Camfield 1971; Camfield and Gough, 1975) or 1-D inversion (Wu, 1968; Schmucker, 1970; Jones and



Hutton, 1979; Jones 1980) can be used to give a conductivity-depth distribution.

#### 2.4 2-D and 3-D Earth models

For 2-D and 3-D Earth models, we still take  $\epsilon$  and  $\mu$  as constants. So we have

$$\nabla \mu = 0 \quad (2.77)$$

$$\text{and} \quad \nabla \cdot \vec{H} = 0 \quad (2.78)$$

Hence (2.9) and (2.10) become, respectively,

$$\nabla^2 \vec{E} = \sigma \mu \partial \vec{E} / \partial t - \nabla (\nabla \cdot \vec{E}) \quad (2.79)$$

$$\text{and} \quad \nabla^2 \vec{H} = \sigma \mu \partial \vec{H} / \partial t - \frac{\nabla \sigma}{\sigma} \times (\nabla \times \vec{H}) \quad (2.80)$$

$$\text{or} \quad \nabla^2 \vec{B} = \sigma \mu \partial \vec{B} / \partial t - \frac{\nabla \sigma}{\sigma} \times (\nabla \times \vec{B}) \quad (2.81)$$

##### 2.4.1 2-D numerical modelling

For the 2-D Earth model the conductivity  $\sigma$  is a function of  $y$  and  $z$ . All quantities are assumed to be independent of the  $x$ -direction. One considers two cases depending on which of the  $\vec{E}$  or  $\vec{H}$  is directed along  $x$  axis. They are called E-polarization or H-polarization, respectively. For E-polarization  $\vec{E}$  is perpendicular to  $\nabla \sigma$ , So (2.9) gives

$$\nabla \cdot \vec{E} = 0 \quad (2.82)$$

Substituting (2.84) into (2.79), We get the diffusion equation for 2-D E-polarization case

$$\nabla^2 \vec{E} = \sigma \mu \partial \vec{E} / \partial t \quad (2.83)$$

$$\text{or} \quad \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = i \omega \sigma \mu E \quad (2.84)$$

For H-polarization (2.80a) is used or it is written in a form

$$\sigma \left[ \frac{\partial^2 B}{\partial y^2} + \frac{\partial^2 B}{\partial z^2} \right] - \frac{\partial \sigma \partial B}{\partial y \partial y} - \frac{\partial \sigma \partial B}{\partial z \partial z} = i \omega \sigma^2 \mu B \quad (2.85)$$

Equations (2.86) and (2.87) can be solved, subject to the appropriate boundary conditions, by a variety of techniques. These techniques are: (1) the finite difference method, (2) the transmission surface analogy, (3) finite element method, (4) the integral equation method.

The finite difference technique was first used by Neves (1957) and was further developed by Jones, Price and Pascoe to study electromagnetic induction in 2-D structures. Jones and Price (1970 and 1971) employed this method to study various 2-D conductivity distributions. Jones and Pascoe (1971) and Pascoe and Jones (1972) gave a general Fortran program for the solution of 2-D induction problems. The method of solution for (2.84) or (2.85) is to represent derivatives as finite differences at a rectangular mesh of grid points and then the finite difference equation are solved by the Gauss-Seidel iterative technique. The finite difference method has been used for field observed data (Camfield, 1973; Rooney and Hutton, 1977; Hutton et al., 1980; and Greenhouse and Bailey, 1981)

The transmission surface analogy arises from the similarity in form between Maxwell's equations governing the electromagnetic field components and transmission surface equations governing current and voltage over a transmission surface with current flowing in two directions. The equivalent transmission surface is approximated by a lumped-circuit network, and the set of equations obtained by applying Kirchhoff's laws to the circuit is solved to give the values of the current and voltage, which correspond to electromagnetic components. This analogy has been used to study 2-D electromagnetic induction by many authors (Madden and Thompson, 1965; Madden and Swift, 1969; Swift, 1967, 1971; Wright, 1969, 1970). Porath and Gough (1971), and Porath, Gough and Camfield (1971) used this method to model conductive structures by using data of their 1968 and 1969 arrays. Peeples and Rankin (1973) used it to model the Precambrian rift in southern Alberta by using MT data.

In the finite element method (Coggon, 1971; Ryu, 1972; Reddy and Rankin, 1973) a 2-D structure is represented by a mesh of finite elements, in each of which the total electromagnetic energy functional is minimized. This leads to a set of linear algebraic equations which are solved by usual procedures.

The integral equation method has been reported (Dmitriev and Zakharov, 1970; Hohmann, 1971). In the integral equation method the solution is in the form of an integral equation, which is reduced to a matrix equation,

and solved numerically for the electric field in the body. The electromagnetic fields at the surface of the half-space are obtained by integrating the half-space Green's functions over the scattering currents. An advantage of the method is that unknown fields need be found only in the anomalous regions. Hence the method are less expensive. However, the integral equation method involves more difficult mathematics. It is preferable for modeling simple geology.

#### 2.4.2 3-D numerical modelling

Of the four techniques employed in 2-D modelling, except the transmission surface analogy, they have all been extended to 3-D electromagnetic induction problems. They are the finite difference method (Lines and Jones, 1973a, b ), the finite element method (Reddy, Rankin and Phillips, 1977; Pridmore, Hohmann, Ward and Sill, 1981) and the integral equation method (Raiche, 1974; Hohmann, 1975; Weidelt, 1975; Meyer, 1976; Vasseur and weidelt, 1977; Dawson and Weaver, 1979; Ting and Hohmann, 1981; Wannamaker et al., 1984). Since 3-D numerical modelling requires large computer storage and consuming computer time, the methods are expensive. Therefore, only very simple geometries of the 3-D models have been studied. They have not so far been applied to array studies.



### 3. THE 1983 ARRAY STUDY

#### 3.1 The Observations

A two-dimensional magnetovariation array of 33 GR magnetometers was operated during the summer of 1983. The locations of the 29 magnetometers which recorded satisfactory data are shown in Figure 3.1. The names and the geographic co-ordinates of the stations are given in APPENDIX I. The array was placed between Thompson, Alberta, in the north and the United States border in the south; and between Revelstoke, British Columbia, in the west and Drumheller, Alberta, in the east. The 33 stations were arranged on five approximately west-east lines. The stations in each line are joined by a thin line, and the lines are numbered 1-5 from south to north. All magnetometers were in operation by 18 July and the first instrument was retrieved on 20 August. The whole field operation, including installation and retrieval, lasted about 12 weeks from 10 June to 29 August.

The purposes of this investigation were three in number. The first aim was to look for a possible continuation of the Northern Rockies (NR) conductor, known to exist between  $54^{\circ}\text{N}$  and  $52^{\circ}\text{N}$ , along the Rockies to the southeast. The NR conductor had been found by the 1980A array (Gough et al., 1982) and was studied in detail by the 1981A array (Bingham et al., 1985). Bingham et al. located the conductor between  $54^{\circ}\text{N}$  and  $52^{\circ}\text{N}$  by analyses of



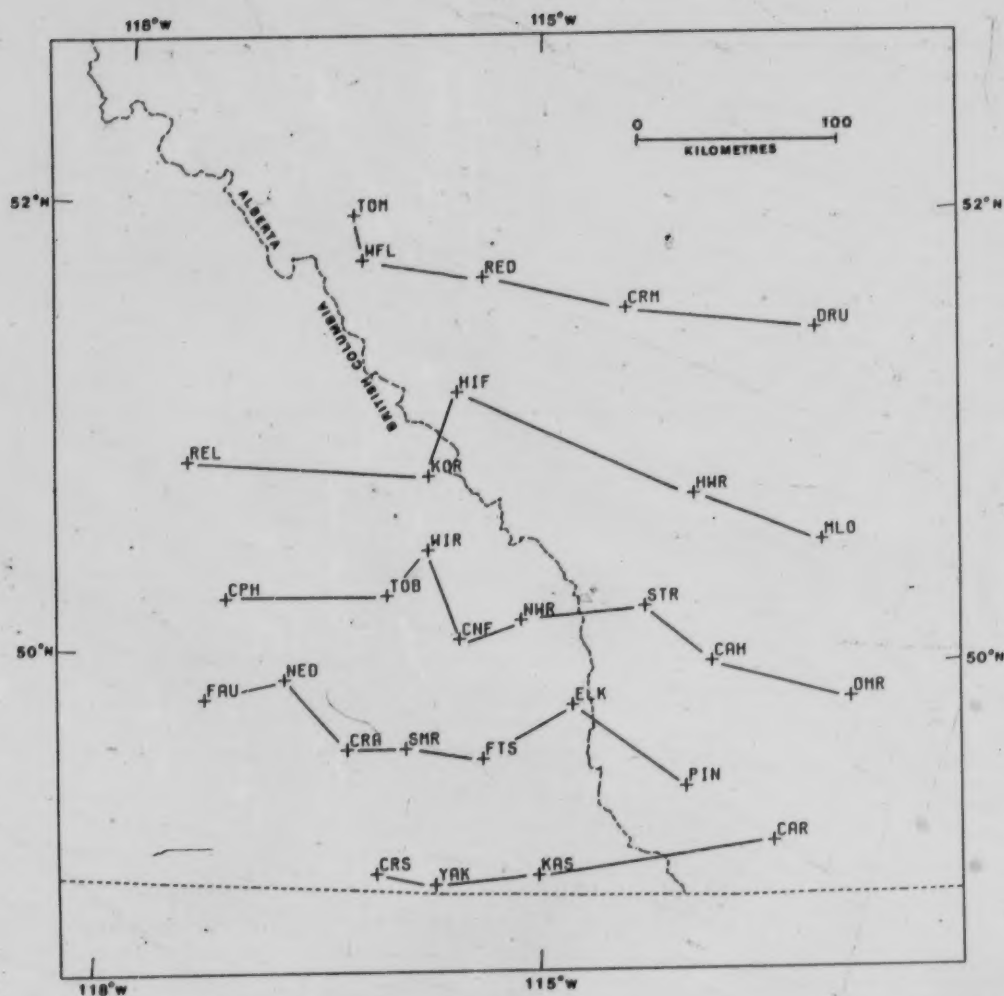


Figure 3.1 Station locations of the 1983 array. The thin lines join stations of five profiles

single-station transfer functions, by means of hypothetical event analysis (see section 4.3 for an explanation of the HEA) and by means of current modelling. The NR conductor lies under the Main Ranges of the Rockies, as was shown in Figure 1.10. It is a significant question for the tectonics of the region, how far this conductor continues along the Rockies to the southeast.

The second objective was to improve the definition of the edge of the Canadian Cordilleran Regional (CCR) conductor along the Kootenay Arc between  $51^{\circ}\text{N}$  and  $49^{\circ}\text{N}$ . It has long been known that the northeastern limit of the CCR conductor is at the Rocky Mountain Trench and main ranges north of about  $51^{\circ}$  and then leaves the Trench to run along Kootenay Lake due south to the United States border (Caner, 1971; Dragert 1973; Dragert and Clarke, 1977; Bingham et al., 1985; Gough, 1986a). But the Kootenay Arc edge of the CCR conductor was not accurately located because of complicated conductive structures in this area.

The third objective was to study was the Southern Alberta-British Columbia (SABC) conductor. This conductor had been detected and discovered by several authors by means of magnetovariation fields (Cochrane and Hyndman, 1970; Dragert and Clarke, 1977; Gough et al., 1982). However it needed a detailed investigation. The 1981B array had been operated in southern and east-central Alberta and the southeastern corner of British Columbia, as shown in Figure 3.2, to study this elongated conductor and its association

with the Precambrian rift proposed by Kanasewich and his colleagues (Kanasewich, 1968; Kanasewich et al., 1969; Clowes and Kanasewich, 1972). The 1983 array was designed to investigate the SABC conductor in relation to a possible extension of the NR conductor, and the edge of the CCR conductor.

### 3.2 Magnetograms

The magnetograms recorded by a magnetovariation array constitute the raw data. Large conductive structures can often be recognized and roughly located by inspection of magnetograms. Over a conductive layer in the Earth, the horizontal components X (northward positive) and Y (eastward positive) are increased, and Z component (downward positive) is reduced; in the limiting case of a perfectly conductive half space, and a high frequency source field, the X and Y components are doubled and Z component is annulled (see section 2.2.1 above). Thus reduced vertical variation fields and enhanced horizontal variation fields will be observed in a region of high conductivity.

Stacked magnetograms for the time intervals 05:30-06:30 UT on 26 July, and 06:30-08:00 UT on 29 July, are shown in Figures 3.3 and 3.4 respectively. Hour marks show universal time (UT). Each stack of magnetograms shows data acquired from a west-east line of stations, with the westernmost station at the top. The lines are numbered 1 to 5 from south to north.

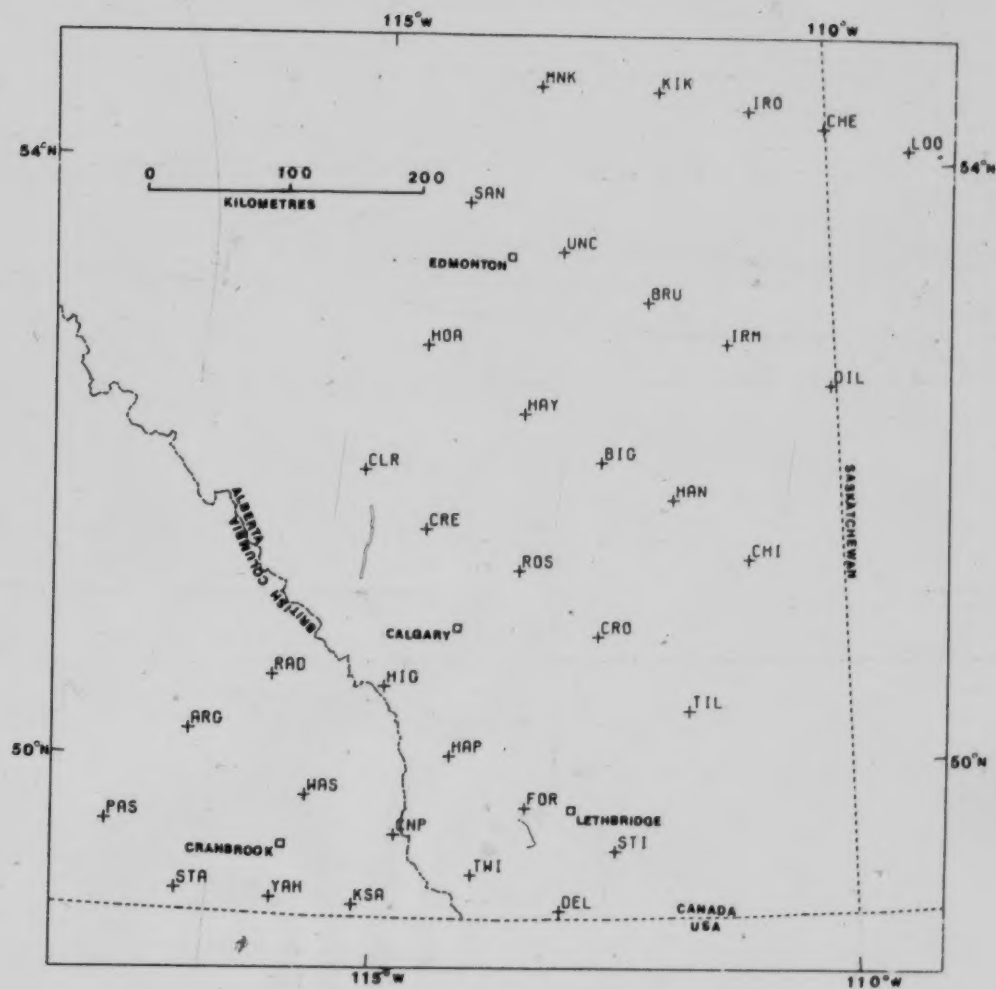


Figure 3.2 Locations of stations in the 1981B array.

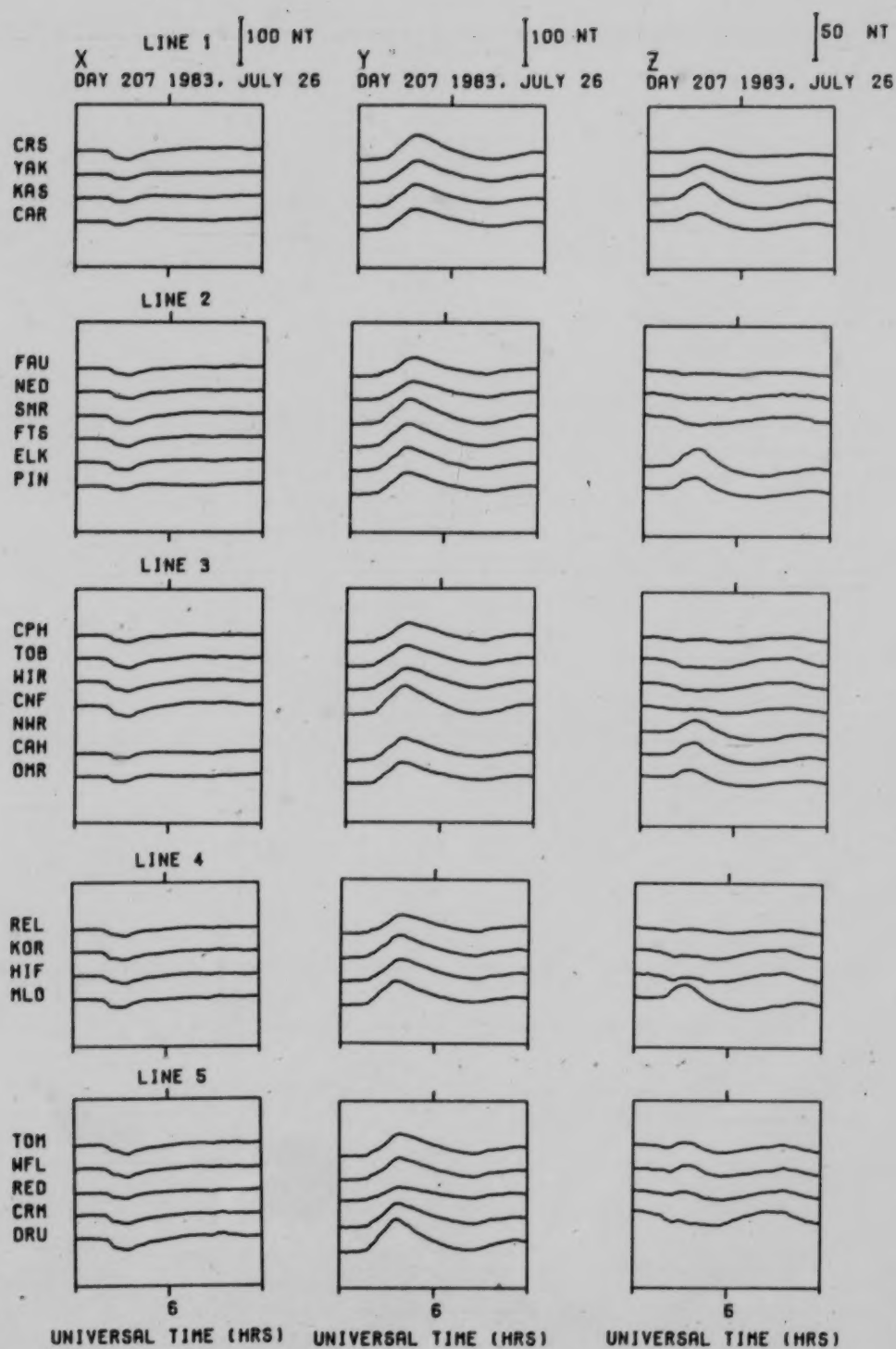


Figure 3.3 Magnetograms for 05:30-06:30 UT, 26 July 1983.



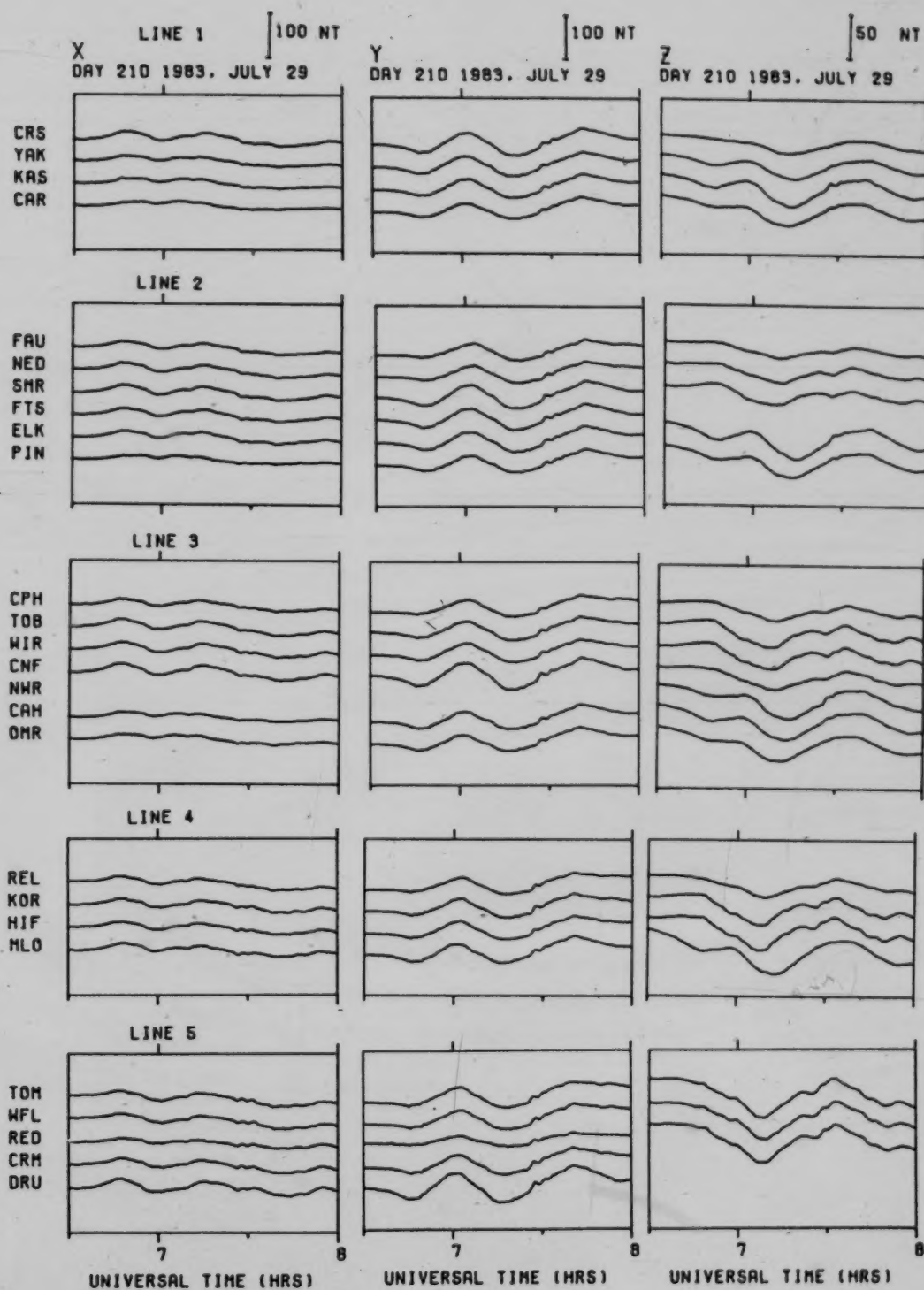


Figure 3.4 Magnetograms for 06:30-08:00 UT, 29 July 1983.

The most obvious feature shown on the magnetograms is the low-Z region in the west and high-Z region in the east. Low amplitudes of the Z variation field characterize the CCR conductor (Hyndman, 1963; Caner et al., 1967 and 1971; Camfield et al., 1971; Gough et al., 1982; and others). From Figures 3.3 and 3.4, We can see that REL (line 4), CPH, TOB, WIR and CNF (line 3), FAU, NED, and SMR (line 2) and CRS (line 1) lie over the low-Z region; and the whole of line 5, MLO (line 4), NWR, CAH, and OMR (line 3), ELK and PIN (line 2), and YAK, KAS and CAR (line 1) are in the high-Z region. Figures 3.3 and 3.4 illustrate the relatively uniform horizontal components ( X and Y ), implying rather uniform source fields for these events.

Now let us look at the limit of the CCR conductor by inspection of the magnetograms. On Figures 3.3 and 3.4, the Z magnetovariation fields obviously change character between REL and KOR (line 4), WIR, CNF and NWR (line 3), SMR and ELK (line 2), CRS and YAK (line 1). A close check of the horizontal component magnetograms shows that Y components at stations of CNF, SMR, and FTS are enhanced. Hence current concentrations with a north-south component must flow beneath these stations.

Unfortunately, we did not obtain data from the Golden station, where the instrument failed. However, Caner et al. (1967 and 1971) have placed the low-Z/high-Z discontinuity on linear array B at Golden (GOL) and on linear array D at McBride (MCB) and Valemount (VAL) (see Figure 1.5). Bingham

et al. (1985) and Gough (1986a) indicated that the northeastern limit of the CCR conductor lay along the Rocky Mountain Trench and Main Ranges north of  $51^{\circ}\text{N}$  and the Northern Rockies (NR) conductor lay under the Rocky Mountains Main Ranges, rather than under the Trench (see Figure 1.10 and 1.20) between  $52^{\circ}\text{N}$  and  $54^{\circ}\text{N}$ .

Magnetovariation arrays often show the presence of anomalously conductive linear structures. The linear structure may produce phase shifts, or even reversals of the sign, of Z variation fields between stations straddling the structures. The transverse horizontal component is enhanced at stations above the structures, but the horizontal component along the structures is not.

Let us examine the SABC conductor by inspection of the magnetograms. From Figures 3.3 and 3.4, we can see that Z variation fields have phase shifts or reversals of the sign between CRS and YAK (line 1), SMR and ELK (line 2), CNF and NWR (line 3), HIF and MLO (line 4). Hence a linear structure must exist between these stations. The horizontal variation fields (X and Y) are enhanced at CRS, SMR, FTS, CNF and DRU, suggesting that a linear structure passes near these stations.

The above inferences from the magnetograms are combined in Figure 3.5 and can be summarized as follows: (1) the eastern limit of the CCR conductor is reasonably well located from latitude  $53.3^{\circ}\text{N}$  (McBride) and runs along the Rocky Mountains Trench and Main Ranges, striking

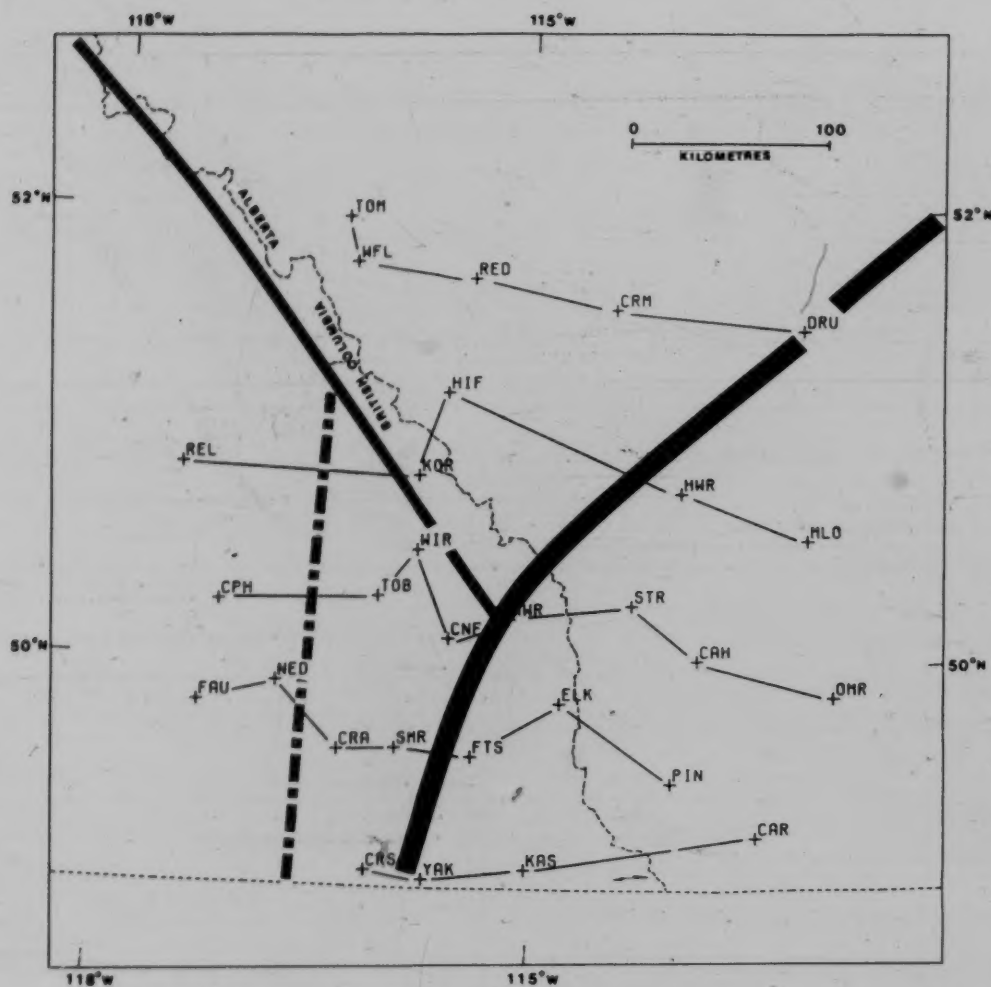


Figure 3.5 Approximate locations of the eastern limit of the CCR conductor and of the SABC conductor, from inspection of magnetograms

southeastward, passing through Valemount and Golden, and somewhere between CNF and NWR at about  $50^{\circ}\text{N}$ . Here it appears to join the SABC conductor, then swings south-southwestward and reaches the United States border between CRS and YAK.

(2) The SABC conductor can be distinguished from the CCR conductor between CNF and NWR and strikes northeastward, passing through Drumheller. (3) The broken line in Figure 3.5 shows the previously proposed north-south discontinuity between low-Z and high-Z regions. The inspection of magnetograms suggests that the eastern limit of the CCR conductor lies nearer the thinner solid line. The thick solid line shows the SABC conductor. However the question of the exact locations of these features will be better defined by more refined techniques considered later in this thesis.

### 3.3 Fourier Transform Anomaly Maps

Contoured maps of Fourier transform (FT) parameters of selected magnetovariation events, at various periods, provide very useful information on the locations of induced currents and anomalously conductive structures in different depth ranges. This technique was introduced by Reitzel et al. (1970) and has been used by many authors (Gough and Ingham, 1983; Bingham et al., 1985 and many others). The algorithm for computing Fourier transtorms is given in APPENDIX II. The technique requires simultaneous recording of an event with a two-dimensional array.



The amplitude spectra are plotted against the period  $T$  for three or four stations near corners of the array. Periods are chosen by the method of Gough and Ingham (1983). Contour maps of amplitude and phase of three components are drawn by means of a computer programme designed for irregularly distributed data. At each period a set of six maps is produced, three for amplitudes and three for phases. Ten sets of FT maps, with the corresponding horizontal field polarization, are shown in Figures 3.6-3.15. Dots indicate the locations of the stations. These maps exhibit the induced field distributions at ten periods of 8.2-170.6 minutes, listed in Table 3.1. The amplitude maps are in arbitrary units while the phases are in minutes. All maps are plotted with 18 contours, so that maps of similar appearance show fields of various ranges. As all FT phase maps for X components show a nearly constant phase, they contain little or no information and are omitted.

The normal field of a magnetovariation event can be defined as the resultant of the field of external currents and currents induced in a model of the regional conductive structure, usually a layered, one-dimensional structure being assumed. Two- or three- dimensional conductive structures will contain anomalous currents induced by the normal field in those structures, and corresponding anomalous fields will be superimposed on the normal field. The polarization ellipses in Figures 3.6-3.15 show only moderate and smooth changes over the whole array, so that

Table 3.1 Data and periods of events for which the Fourier transform maps are drawn

| Event | Date         | Period<br>(mins) | Length of data sequence |
|-------|--------------|------------------|-------------------------|
| 08F   | July 30 1983 | 8.2              | 01:00-02:00             |
| 03A   | July 24      | 15.8             | 18:30-19:30             |
| 08D   | July 29-30   | 21.5             | 23:00-02:30             |
| 03C   | July 24      | 24.4             | 19:30-21:00             |
| 03A   | July 24      | 27.5             | 18:30-19:30             |
| 08A   | July 29-30   | 48.3             | 23:00-07:30             |
| 08E   | July 30      | 52.0             | 00:00-01:30             |
| 08A   | July 29-30   | 111.7            | 23:00-07:30             |
| 08B   | July 29-30   | 136.5            | 23:00-04:00             |
| 01A   | July 18      | 170.6            | 01:00-06:00             |

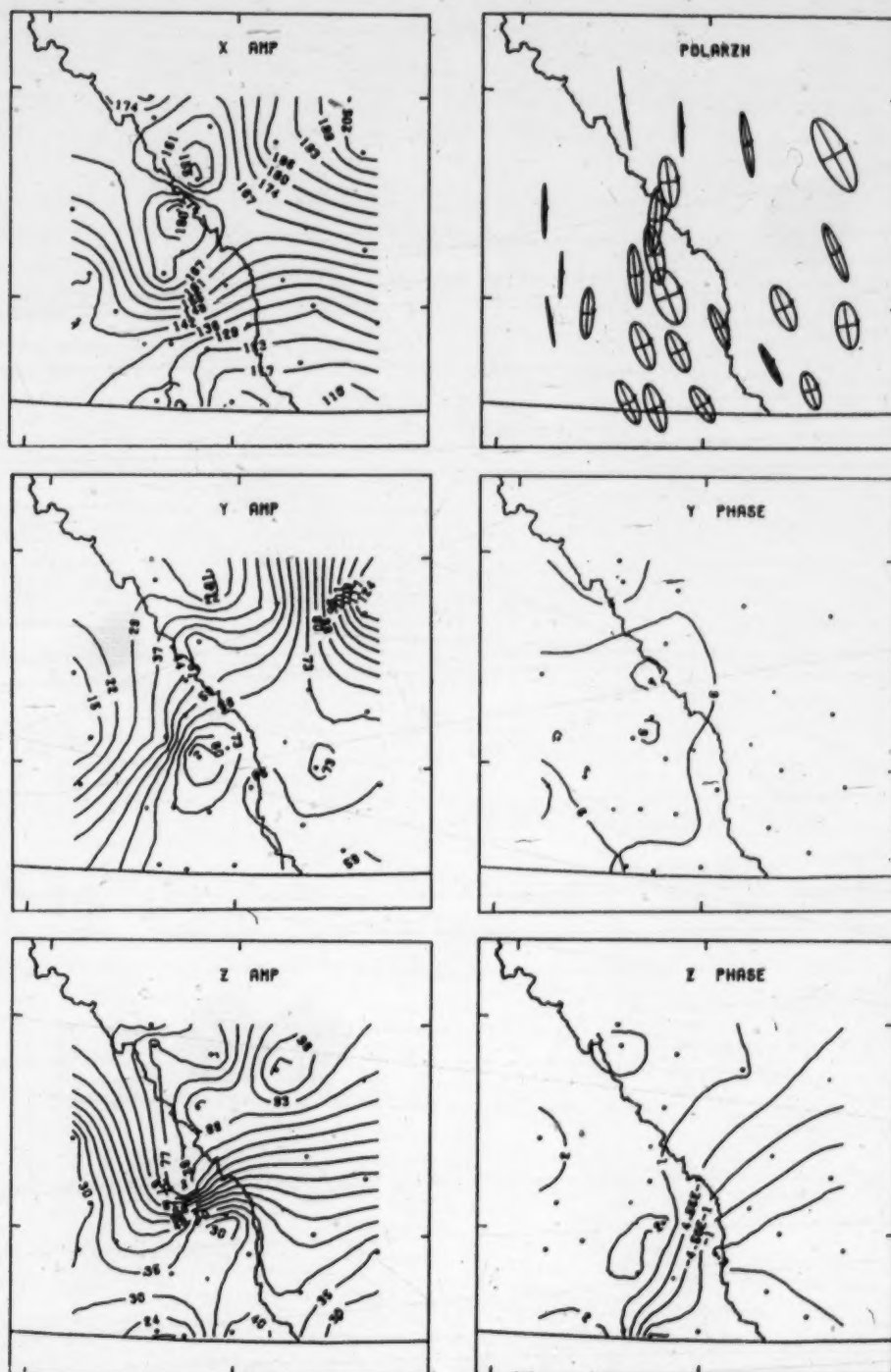


Figure 3.6 Fourier transform anomaly maps and horizontal field polarization at  $T = 8.2$  min for a data sequence 01:00-02:00 UT, 30 July 1983.

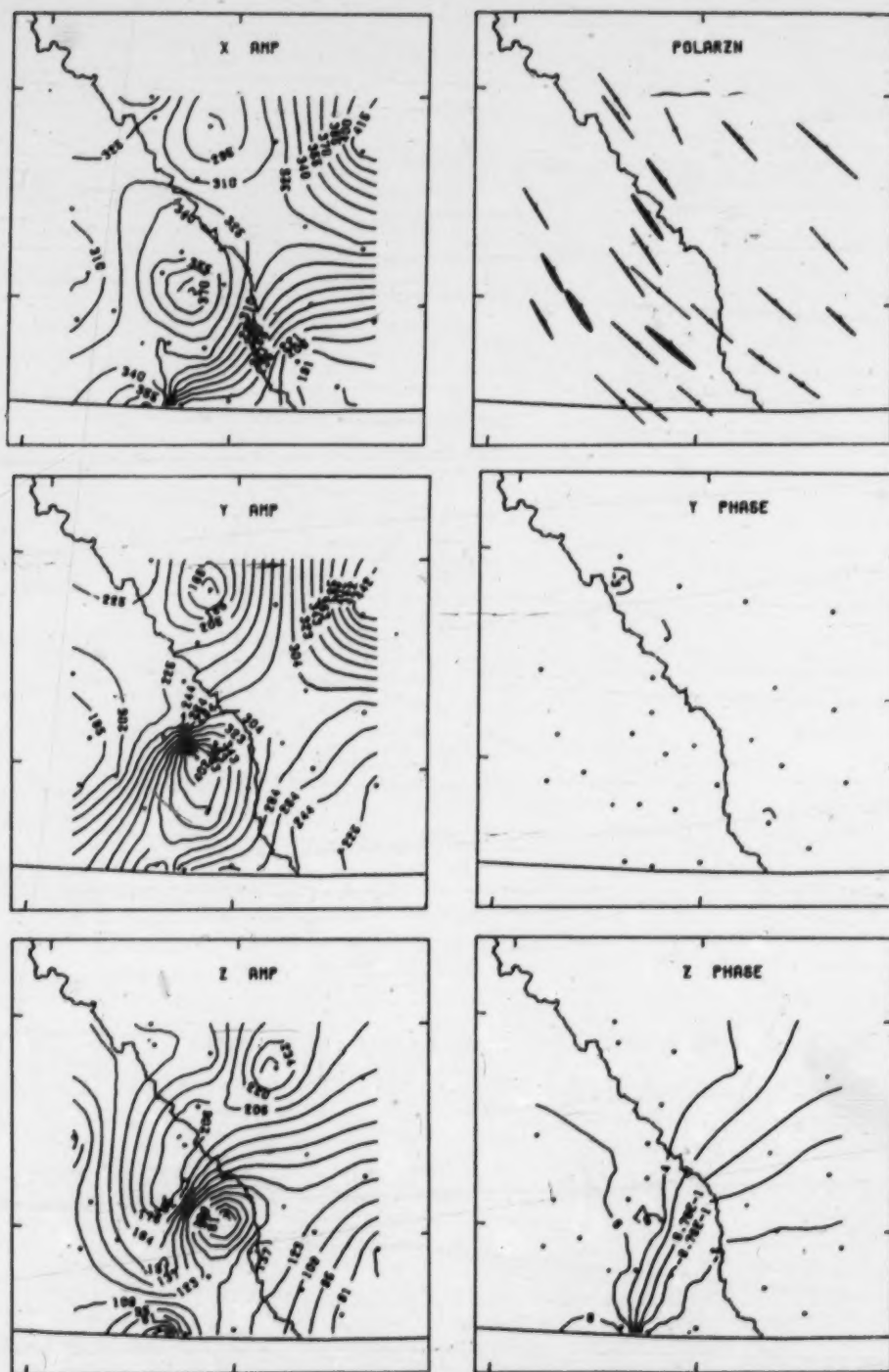


Figure 3.7 Fourier transform anomaly maps and horizontal field polarization at  $T = 15.8$  min for a data sequence 18:30-19:30 UT, 24 July 1983.



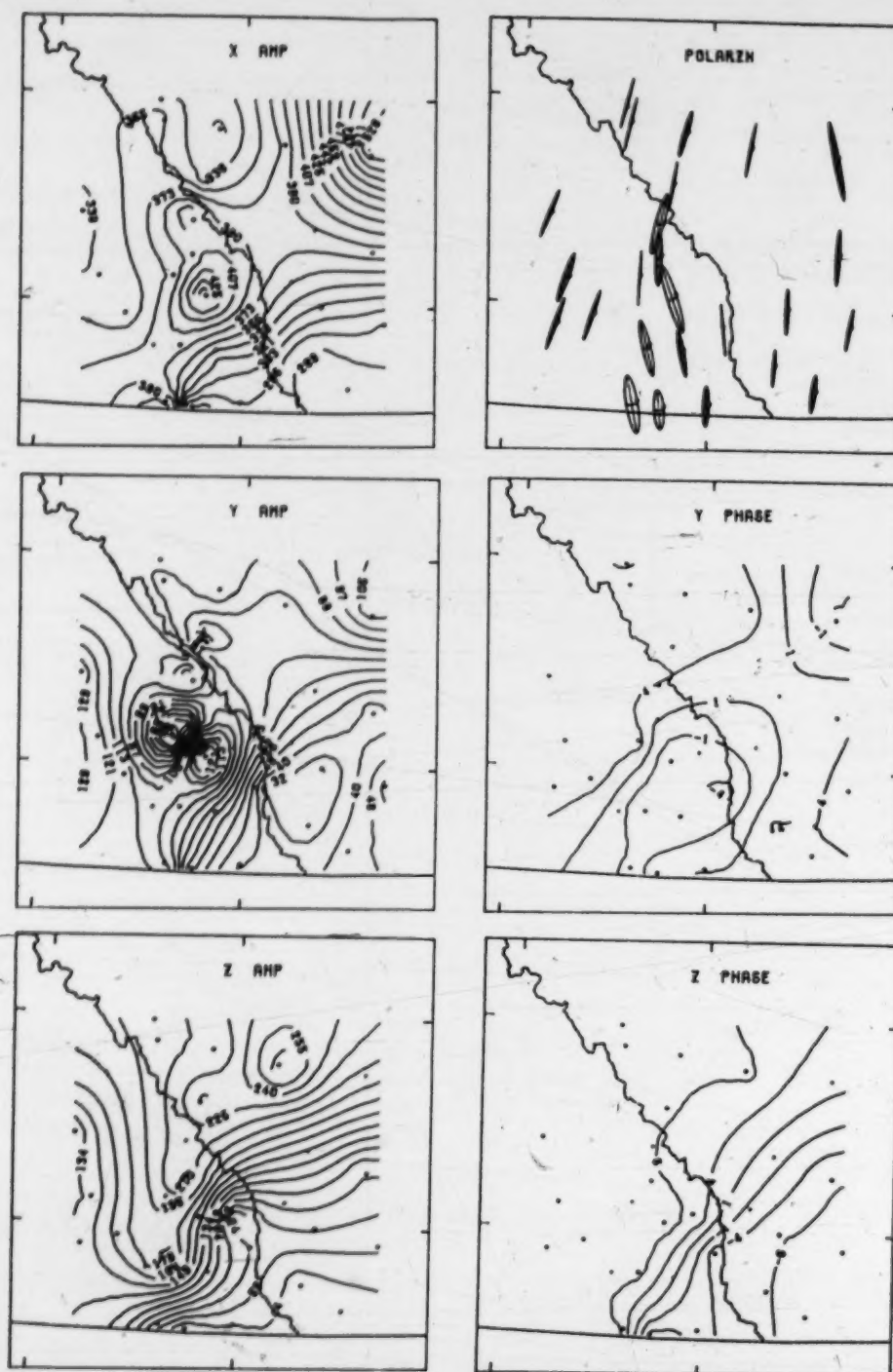


Figure 3.8 Fourier transform anomaly maps and horizontal field polarization at  $T = 21.5$  min for a data sequence 23:00-02:30 UT, 29-30 July 1983.



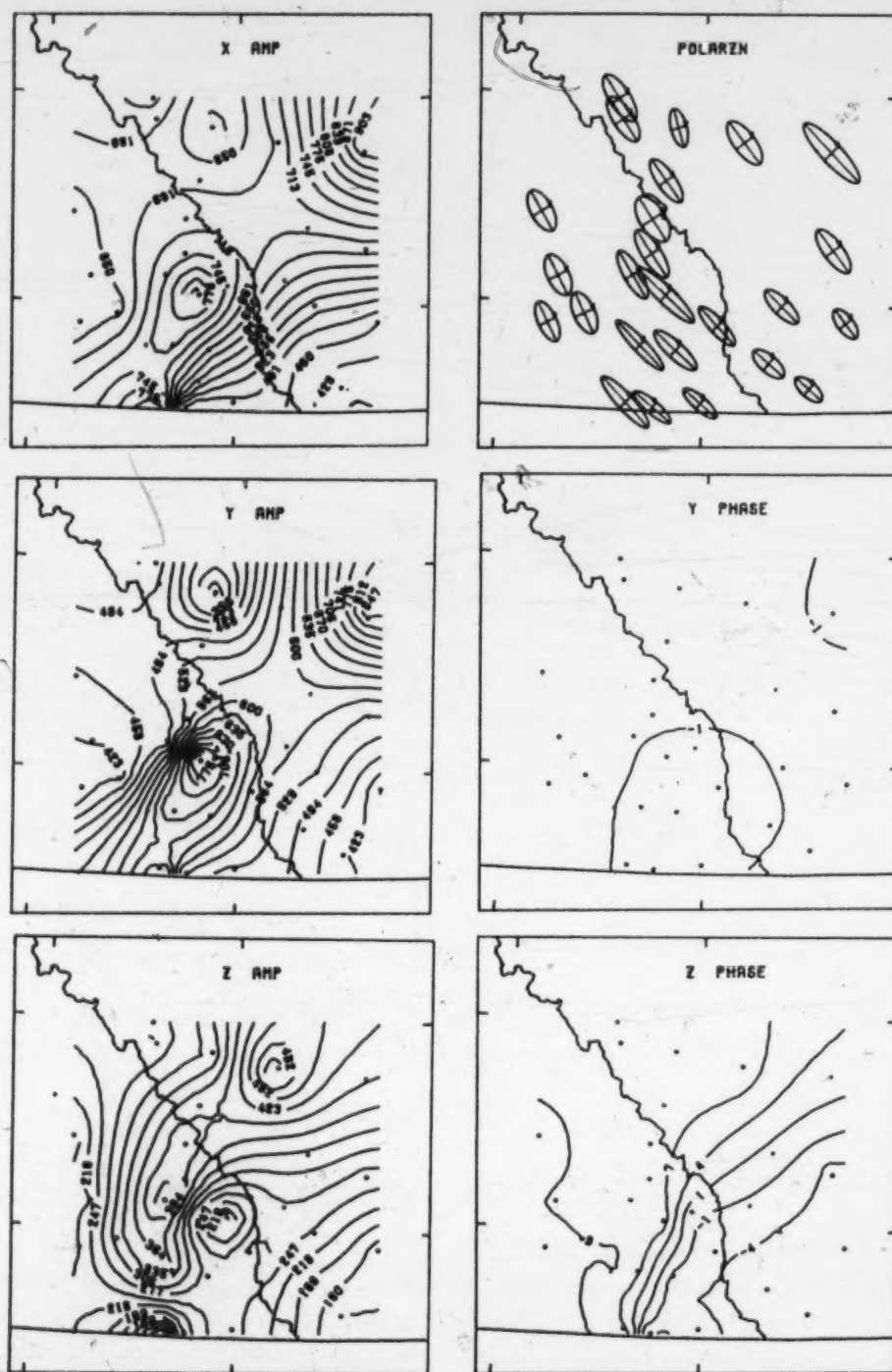


Figure 3.9 Fourier transform anomaly maps and horizontal field polarization at  $T = 24.4$  min for a data sequence 19:30-21:00 UT, 24 July 1983.

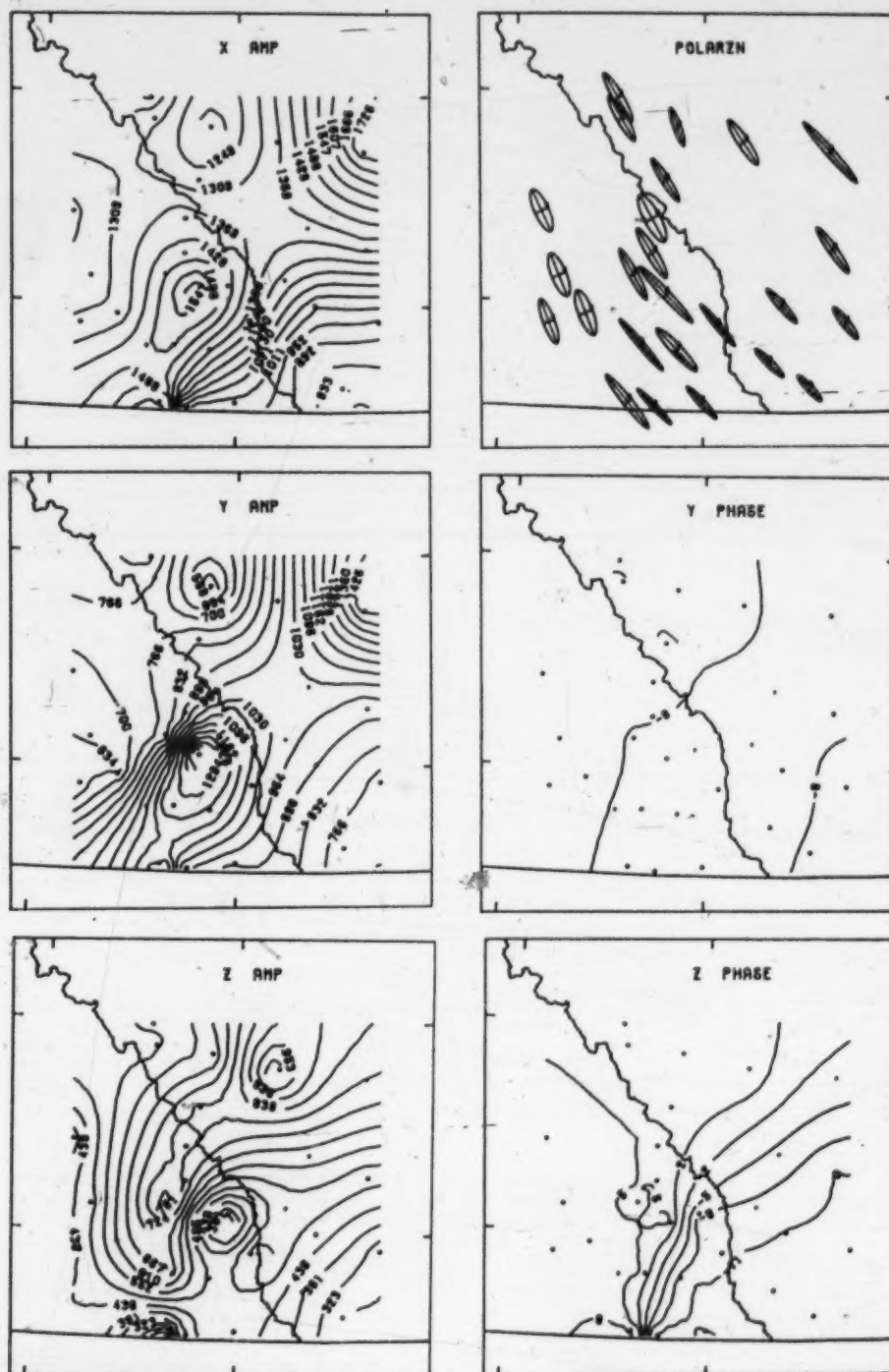


Figure 3.10 Fourier transform anomaly maps and horizontal field polarization at  $T = 27.5$  min for a data sequence 18:30-19:30 UT, 24 July 1983.

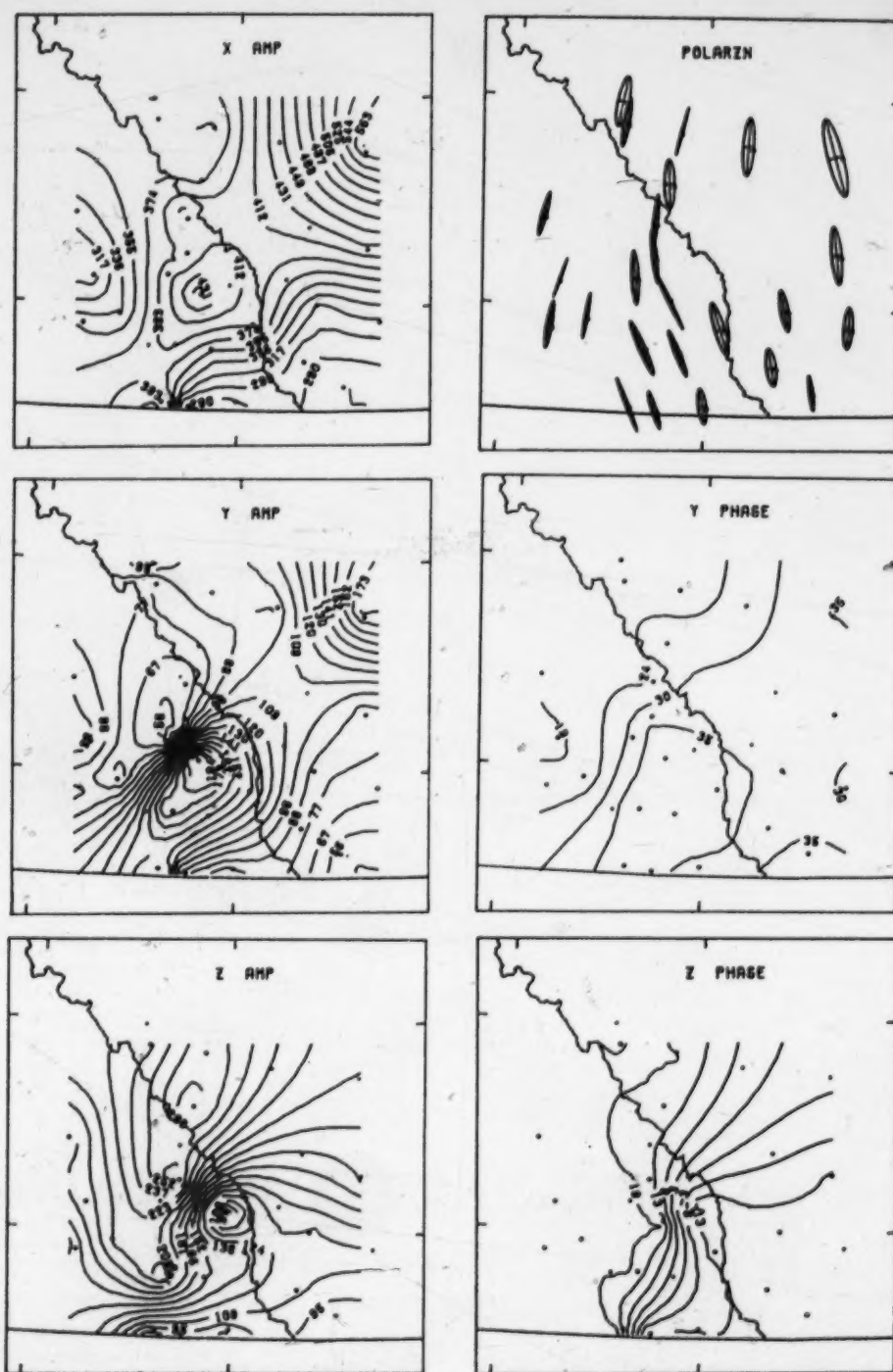


Figure 3.11 Fourier transform anomaly maps and horizontal field polarization at  $T = 48.3$  min for a data sequence 23:00-07:30 UT, 29-30 July 1983.

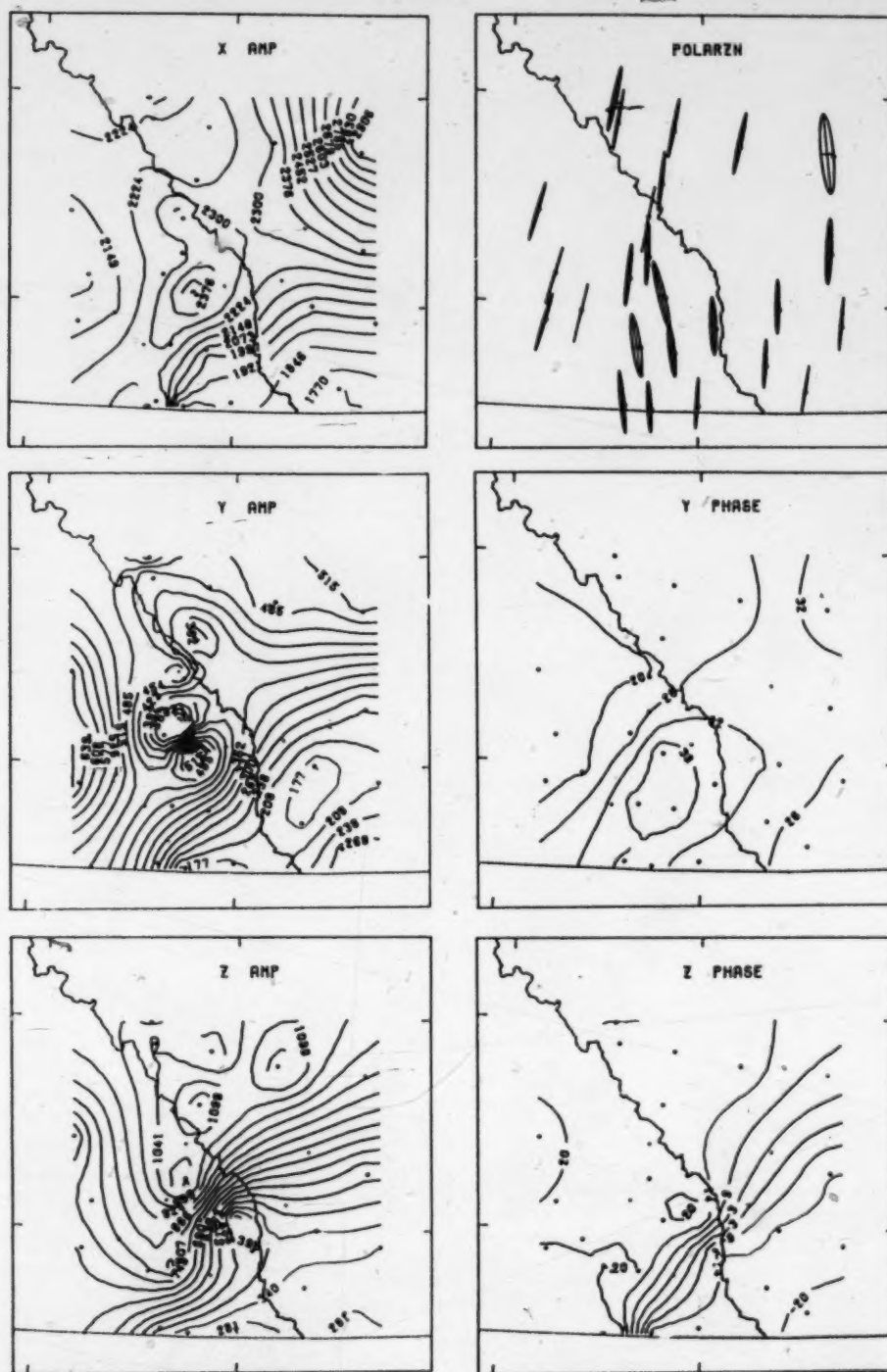


Figure 3.12 Fourier transform anomaly maps and horizontal field polarization at  $T = 52.0$  min for a data sequence 00:00-01:30 UT, 30 July 1983.



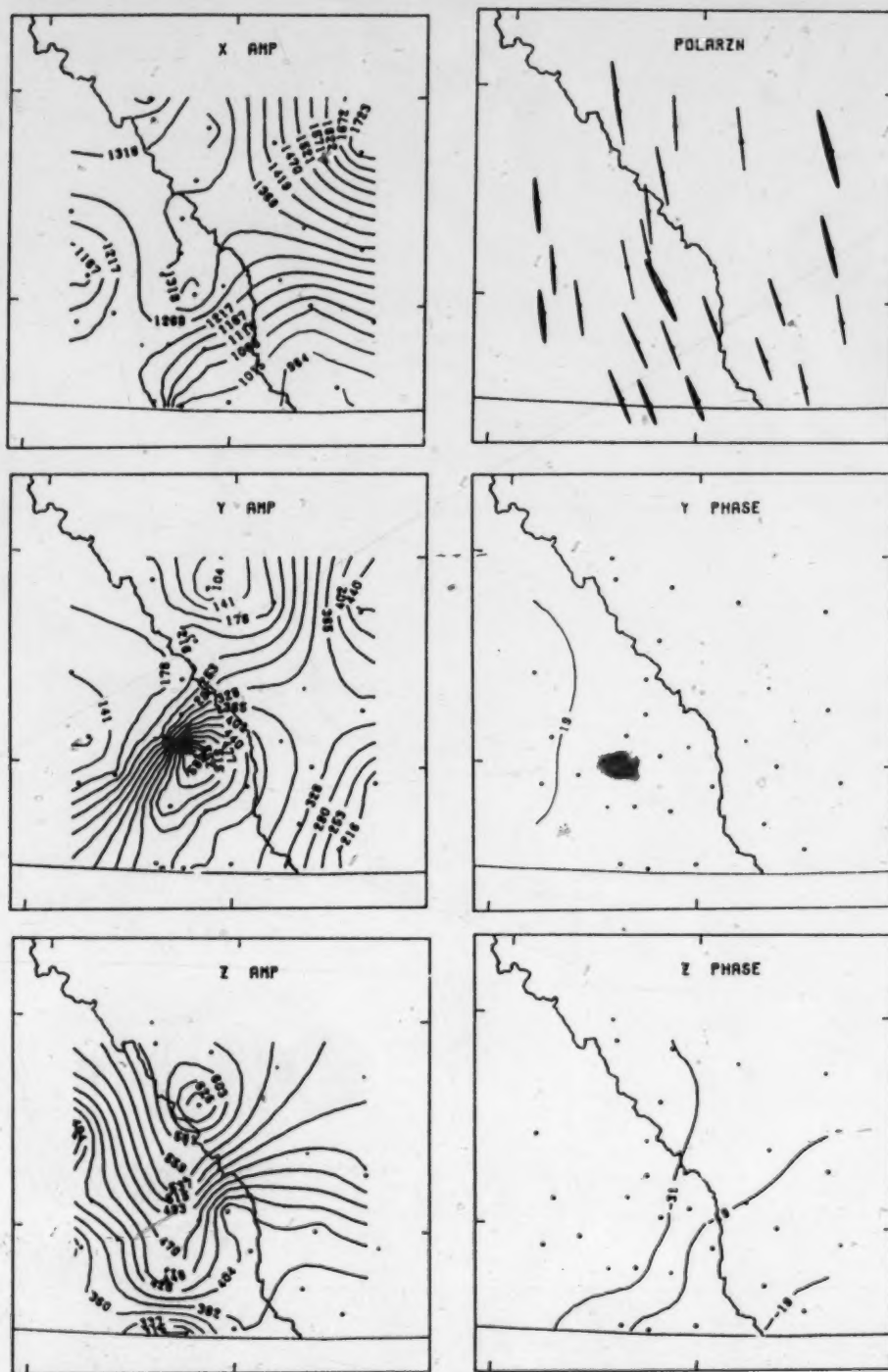


Figure 3.13 Fourier transform anomaly maps and horizontal field polarization at  $T = 111.7$  min for a data sequence 23:00-07:30 UT, 29-30 July 1983.





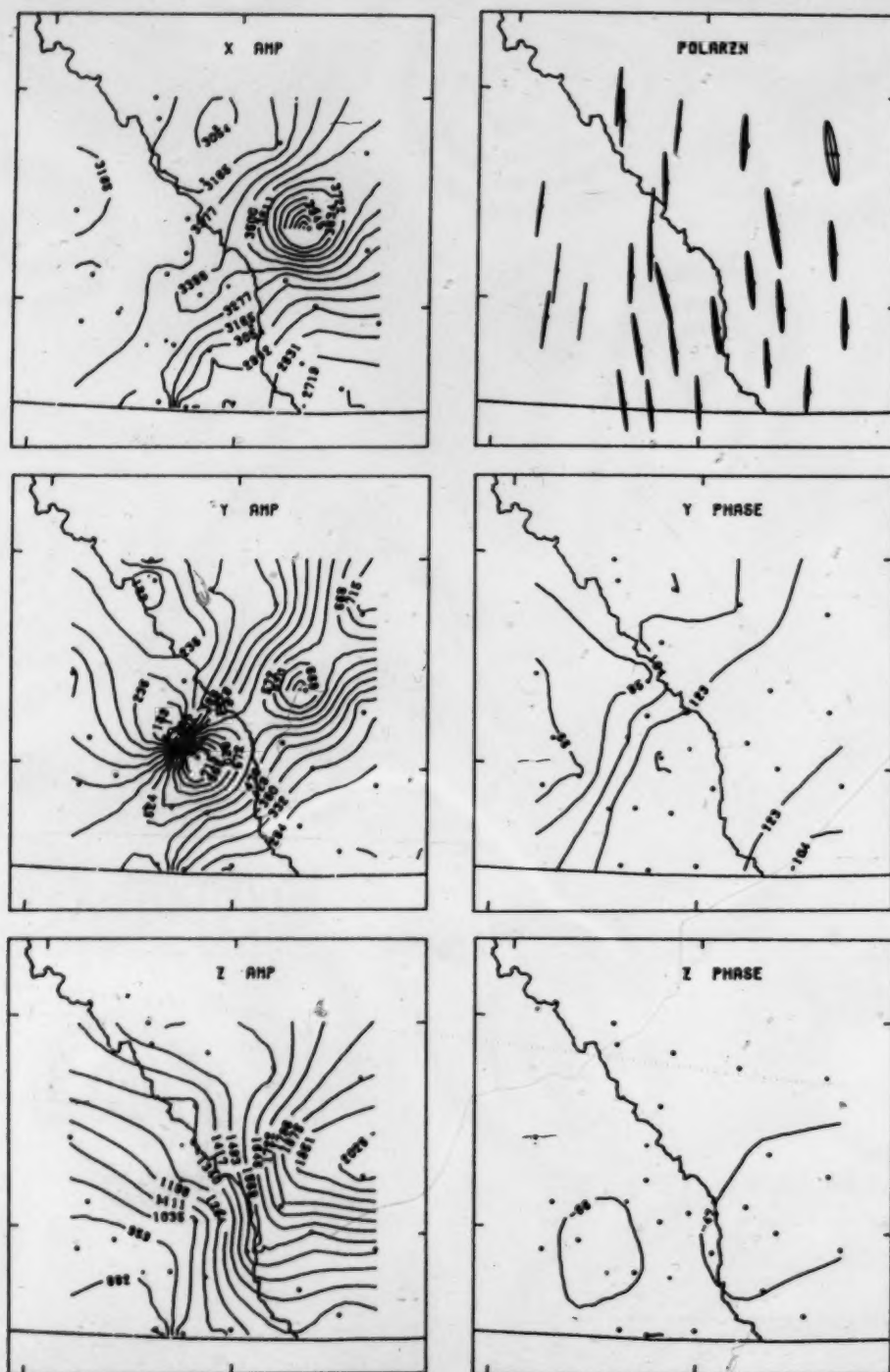


Figure 3.15 Fourier transform anomaly maps and horizontal field polarization at  $T = 170.6$  min for a data sequence 01:00-06:00 UT, 18 July 1983.

for the events chosen, the normal fields are fairly uniform over the array and most features of the maps are attributable to anomalous currents.

The most prominent and consistent anomaly is shown on the phase maps of the Z component. It is almost identical for periods 8.2-52.0 mins and for various polarizations, but it diminishes at periods 111.7 and 136.5 mins and disappears at period 170.6 mins. This elongated anomaly in Z phase runs northeast from Creston (CRS) between CRA and ELK, TOB and CAH, HIF and MLO. Its centre line lies close to CRS, YAK, CNF, NWR and DRU, as noted in the magnetograms. This anomaly is produced by currents induced in the Southern Alberta-British Columbia (SABC) conductor.

Since the 1983 array stations DRU, HWR and STR failed to record, and are very important for locating the SABC anomaly, a set of FT maps at period 85.5 mins of the 1981B array is presented (Figure 3.16). These maps are derived from a data sequence 22:20-00:20 UT, 07-08 October, 1981. The number of contours again is 18. Again the phase map for X component is omitted, for it is a blank map. The basic features of these FT maps are consistent with those of the 1983 array. The agreement of the Z phase maps is excellent. Furthermore, the phase map of 1981B array extends the SABC anomaly to the Saskatchewan border. The SABC conductor is also well shown in the Y amplitude map of Fig. 3.16.

The Z amplitude maps in Figs. 3.6-3.15 show very complicated features due to mixing of the westward





attenuation, the southward attenuation and Southern Alberta-British Columbia anomaly, with small local anomalies superimposed. However these features persist at various periods. The array may be divided into three zones: a northwestern zone, the SABC conductor zone and a southeastern zone. The northwestern zone mainly shows the westward attenuation which has been studied by many authors; the southwestern zone mainly shows the southward attenuation, which has also been observed by Camfield et al. (1971) and by Gough et al. (1982); the SABC conductor zone mainly shows the SABC anomaly with superimposition of small local anomalies at CRS, CNF and NWR, and CRM. The features mentioned above change with period. The small local anomalies at CRS, CNF and NWR, and CRM disappear and a new local anomaly at MLO occurs at period 136.5 mins (Figure 3.14). The SABC anomaly disappears at ~170.6 mins and the whole array shows the westward and southward attenuations only (Figure 3.15).

Most of the Y phase maps show little or no information, but a few of them enclose an anomaly, which locates in the SABC conductor (Figures 3.8, 3.9, 3.11 and 3.12).

The SABC conductor produces considerable anomalies in horizontal components. Two striking anomalies in the amplitudes of X and Y occur at CNF and DRU. The Z component was not recorded at DRU, but Z amplitudes are reduced at CNF and NWR and a minimum occurs at NWR. The horizontal components were not recorded at NWR, but NWR is very close



to CNF. It seems clear that the current flows beneath stations CNF or NWR and DRU. At period 170.6 mins all three components are recorded at HWR, at which no other event was recorded, and shows maximum in both horizontal components. Thus the SABC anomalies are mainly two-dimensional, but with some three-dimensional features superimposed. In addition, the amplitude of the X component has a strong gradient, increasing northeastward, in all of Figures 3.6-3.15. This is regarded as an increase of the normal field as the auroral zone is approached.

## 4. TRANSFER FUNCTIONS AND HYPOTHETICAL EVENT ANALYSES

### 4.1 Single-Station Transfer Functions

The observed magnetic field at any point can be considered as the sum of a normal field  $\vec{H}_n(t)$  and an anomalous field  $\vec{H}_a(t)$ ,

$$\vec{H}(t) = \vec{H}_n(t) + \vec{H}_a(t) \quad (4.1)$$

The normal field is defined as the sum of contributions from the external field and that part of the internal field which is due to the regional conductivity structure, often taken to be one-dimensional. The anomalous field is considered as being due to currents in a two- or three-dimensional conductivity structure. Because Maxwell's equations are linear and the Earth can be regarded as a linear system (Rokityansky, 1982, p.28), the anomalous field (output) can be expressed as the convolution integral of the normal field (input) with an impulse response function. The process can be symbolically written

$$\begin{array}{ccccc} \vec{H}_n(t) & \xrightarrow{\quad \quad \quad} & \vec{W}(t) & \xrightarrow{\quad \quad \quad} & \vec{H}_a(t) \\ & & \text{Earth} & & \\ \text{(input)} & & \text{(linear system)} & & \text{(output)} \end{array}$$

where  $\vec{W}(t)$  is the impulse response function. Thus one has

$$\begin{aligned} \vec{H}_a(t) &= \vec{W}(t) * \vec{H}_n(t) \\ &= \int_{-\infty}^{\infty} \vec{W}(\tau) \vec{H}_n(t-\tau) d\tau \end{aligned} \quad (4.2)$$

By the theorem of convolution, one has

$$\vec{H}_a(\omega) = \vec{W}(\omega) \vec{H}_n(\omega)$$

$$\text{or } \vec{H}_a(T) = \vec{W}(T)\vec{H}_n(T) \quad (4.3)$$

Since  $\vec{H}_a$  and  $\vec{H}_n$  are vectors,  $\vec{W}(\omega)$  and  $\vec{W}(T)$  are tensors.  $\vec{W}(\omega)$  and  $\vec{W}(T)$  are usually called the induction matrix, or the matrix of the transfer function.

Equation (4.3) can be written in terms of the components (X, Y, Z) of the fields as

$$X_a = W_{xx}X_n + W_{xy}Y_n + W_{xz}Z_n \quad (4.4)$$

$$Y_a = W_{yx}X_n + W_{yy}Y_n + W_{yz}Z_n \quad (4.5)$$

$$\text{and } Z_a = W_{zx}X_n + W_{zy}Y_n + W_{zz}Z_n \quad (4.6)$$

To solve equations (4.4)-(4.6) for the elements of the matrix  $\vec{W}(T)$  it is necessary to estimate the normal  $\vec{H}_n(T)$ . When the normal field  $\vec{H}_n(T)$  cannot be estimated, some simplifying assumptions are needed. Banks (1973) and Gough and Ingham (1983) have discussed these assumptions and their validity. These assumptions may be quite safe in middle latitudes.

If (1)  $Z_n$  is small, and the terms  $W_{xz}$ ,  $W_{yz}$  and  $W_{zz}$  can be ignored and  $Z_n + Z_a \approx Z_a \approx Z$ ; (2)  $X_n \gg X_a$  and  $Y_n \gg Y_a$ , i.e.  $X_n \approx X$  and  $Y_n \approx Y$ , one has

$$Z \approx W_{zx}X + W_{zy}Y \quad (4.7)$$

which is usually written in the form

$$Z \approx AX + BY \quad (4.8)$$

in which all the fields are the measured values, and A and B are known as single station transfer functions because they can be estimated from magnetovariation fields recorded at

only one station. In equation (4.8) all quantities are complex.

Equation (4.8) is more correctly written as

$$Z = AX + BY + \epsilon \quad (4.9)$$

where  $\epsilon$  is the error term due to assumptions (1) and (2). A and B may be estimated from observational data (Everett and Hyndman, 1967; Schmuicker, 1970; Gough and Ingham, 1983),

Minimizing  $\sum \epsilon \epsilon^*$  with respect to the real and imaginary parts of A and B, one finds:

$$\sum ZX^* = A \sum XX^* + B \sum YX^* + \sum \epsilon X^* \quad (4.10)$$

$$\text{and} \quad \sum ZY^* = A \sum XY^* + B \sum YY^* + \sum \epsilon Y^* \quad (4.11)$$

Here the asterisk indicates the complex conjugate and the summation is over a number n of magnetovariation events. Provided  $\epsilon$  does not correlate with X and Y and n is large enough, the terms  $\sum \epsilon X^*$  and  $\sum \epsilon Y^*$  can be taken as zero, and (4.10) and (4.11) lead to

$$A = \frac{\sum ZX^* \sum YY^* - \sum ZY^* \sum YX^*}{\sum XX^* \sum YY^* - \sum YX^* \sum XY^*} \quad (4.12)$$

$$\text{and} \quad B = \frac{\sum ZY^* \sum XX^* - \sum ZX^* \sum XY^*}{\sum XX^* \sum YY^* - \sum YX^* \sum XY^*} \quad (4.13)$$

Similar expressions exist for the remaining transfer functions involved in equations (4.4) and (4.5). However, it has been found difficult to obtain their stable estimates (Cochrane and Hyndman, 1970).

Parkinson (1959) first proposed a relationship,

$$\Delta Z = A_p \Delta X + B_p \Delta Y \quad (4.14)$$

where  $\Delta X$ ,  $\Delta Y$  and  $\Delta Z$  are actual in-phase changes of observed magnetovariation fields over some successive time intervals, and  $A_p$  and  $B_p$  are real constants. Equation (4.14) defines a plane in three-dimensional real space, which is called the "preferred plane" or "Parkinson plane" (Parkinson, 1959; 1962 and 1964). A downward unit arrow was drawn normal to the preferred plane. Its horizontal projection was called the Parkinson arrow, which we denote by  $A_p$ . The Parkinson arrow points toward a region of anomalously high electrical conductivity. The Parkinson arrow, derived in this manner, does not contain any information about the phase relationships of the fields.

Wiese (1962) plotted the ratios  $Y/Z$  as a function of the ratios  $X/Z$ .  $X$ ,  $Y$  and  $Z$  are components of the magnetovariation fields for a specific period. He found that points lay approximately along a straight line, which was called the "Wiese line". The equation of the Wiese line can be written as

$$Z = a_v X + b_v Y \quad (4.15)$$

where  $a_v$  and  $b_v$  are real constants.

Wiese (1962) defined an induction arrow by a vector with components  $+a_v$  along the positive  $X$  direction and  $+b_v$  along the positive  $Y$  direction. This is called the Wiese induction arrow, which we denote by  $A_v$ .



Schmucker (1970) introduced an induction arrow representation for the transfer functions. Since  $A$  and  $B$  are complex, they can be written as

$$A = A_r + A_i i \quad (4.16)$$

and  $B = B_r + B_i i \quad (4.17)$

Let  $\vec{j}$ ,  $\vec{k}$  be Cartesian unit vectors toward geographic north and geographic east at a given station. The induction arrows are defined as

$$\vec{C}_r = -A_r \vec{j} - B_r \vec{k} \quad (\text{in-phase}) \quad (4.18)$$

and  $\vec{C}_i = -A_i \vec{j} - B_i \vec{k} \quad (\text{quadrature phase}) \quad (4.19)$

at any particular frequency. Here we call the in-phase arrow the "Schmucker arrow", denoted by  $A_s$  instead of  $\vec{C}_r$ .

The negative orientation of both arrows has been chosen for reasons which we now explain. The in-phase arrows are in the same sense as the Parkinson arrows, which point toward regions of high electrical conductivity in which the subsurface current flows in phase with the normal fields. A recent study by Lilley and Arora (1982) points out that the signs of the quadrature-phase coefficients change, depending on whether a time dependence of  $e^{+i\omega t}$  or of  $e^{-i\omega t}$  represents the magnetovariation fields. Quadrature-phase arrows must be reversed for a time dependence  $e^{+i\omega t}$  (such as in this thesis) but should not for a time dependence  $e^{-i\omega t}$  in order that the quadrature-phase arrows point toward good conductors.

The in-phase arrow will be larger than the quadrature-phase arrow when larger and deeper conductive

bodies are involved and the quadrature arrow will be large when near-surface layers are concerned. Conductive structures in the Earth show both inductive reactance and resistance, giving rise to both in-phase and quadrature-phase arrows. In the highly reactive case the in-phase arrow will dominate, whereas in the highly resistive case the quadrature-phase arrow will dominate. Sedimentary rocks near the surface often have resistance larger than reactance while in deep rocks the self-inductance is larger than resistance (Gough et al., 1973, 1974; Lilley and Arora, 1982).

Recently a few papers have appeared discussing the relationships between various induction arrows (Gregori and Lanzerotti, 1980; Jones, 1981 and Wolf, 1982). Gregori and Lanzerotti (1980) expressed the equation of the Parkinson plane as

$$\alpha_1 X + \alpha_2 Y + \alpha_3 Z = 0 \quad (4.20)$$

instead of (4.14), where  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are direction cosines.

If  $\beta$  is the tilt of the Parkinson plane with respect to the horizontal plane, then  $\beta = \alpha_3$ , and the length of the Parkinson arrow  $A_p$  is given (see Figure 4.1) by

$$|A_p| = \sin \beta \quad (4.21)$$

Equation (4.20) can be rewritten as

$$\frac{\alpha_1}{\alpha_3} X + \frac{\alpha_2}{\alpha_3} Y + Z = 0 \quad (4.22)$$

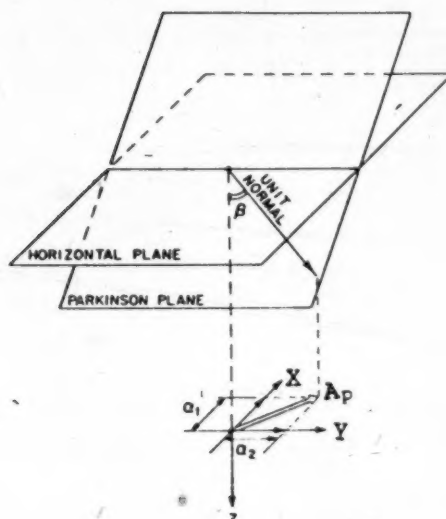


Figure 4.1 Definition of the Parkinson arrow  $A_p$ . Modified from Gregori and Lanzerotti (1980)

A comparison between (4.15) and (4.22) gives  $a_v = -\alpha_1/\alpha_3$  and  $b_v = -\alpha_2/\alpha_3$ . Hence we can calculate the length of the Wiese arrow as follows

$$|A_v| = (a_v^2 + b_v^2)^{1/2} = \left(\frac{1 - \alpha_3^2}{\alpha_3^2}\right)^{1/2} = \left(\frac{\sin^2 \beta}{\cos^2 \beta}\right)^{1/2} = \tan \beta \quad (4.23)$$

where we have used the relation:

$$\alpha_1 + \alpha_2 + \alpha_3 = 1 \quad (4.24)$$

We note in Figure 4.1 that the Parkinson arrow  $A_p$  has components  $\alpha_1$  along X and  $\alpha_2$  along Y, but the Wiese arrow  $A_v$  has components  $-\alpha_1/\alpha_3$  along X and  $-\alpha_2/\alpha_3$  along Y. Hence the Parkinson arrow is oppositely directed to the Wiese arrow. So the Parkinson arrow and the Wiese arrow can be written in terms of one another by the expressions

$$A_p = -A_v / (1 + |A_v|^2)^{1/2} \quad (4.25)$$

$$A_v = -A_p / (1 + |A_p|^2)^{1/2} \quad (4.26)$$

Comparing (4.8) to (4.22) and considering definitions of the Schmucker arrow and the Wiese arrow, we can obtain the relation

$$A_s = -A_v \quad (4.27)$$

and the relationships between the Schmucker arrow and the Parkinson arrow can be derived using (4.25) and (4.26).

Plots of the induction arrows at stations of the 1983 array, from single-station transfer functions for three

periods, are shown in the upper halves of Figures 4.2-4.4. In these figures the in-phase arrows have heads; the quadrature-phase arrows do not. From 6 to 16 independent magnetovariation events of well distributed polarizations were available at the various stations. As an example, Figure 4.5 shows the azimuths of major axes of polarization ellipses for the events at period 49.0 minutes at stations REL and MLO.

In the area of this investigation, the effects of currents induced in the CCR and SABC conductors are superposed to produce the induction arrows at all stations. First of all, let us take a look at the in-phase arrows in the upper parts of Figures 4.2-4.4. At stations YAK, ELK, MLO and all stations east of a line joining these stations, the real induction arrows point northwestward, that is, toward the CCR and SABC conductors. At stations CRS, SMR, CNF, CRM and stations further west, the real induction arrows point in varying directions in the southwest quadrant. Currents in the SABC conductor flow between these two groups of stations. The induction arrows at  $T = 8.5$  min, at stations CRS, SMR and CNF, become much smaller, indicating that the current in the SABC conductor flows near these stations. The orientation of induction arrows west of the SABC conductor turns counter-clockwise with increase of period, but no such rotation occurs east of the SABC conductor or at CRS. This indicates that the CCR conductor is varying with depth.



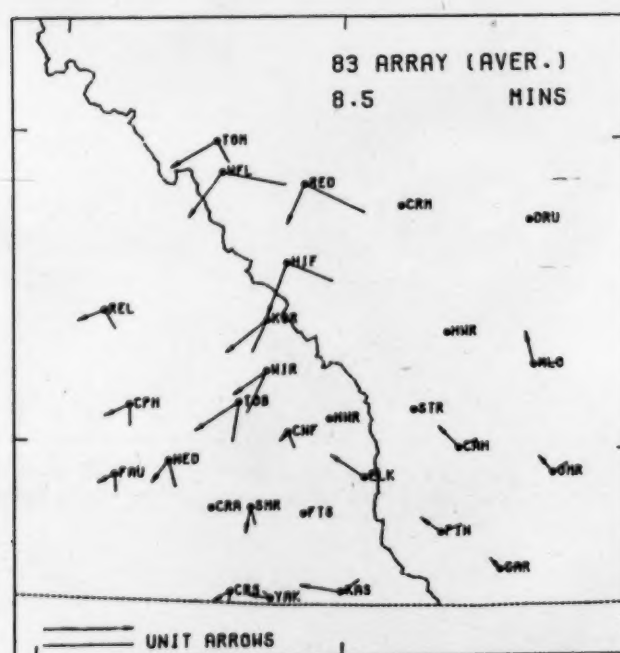
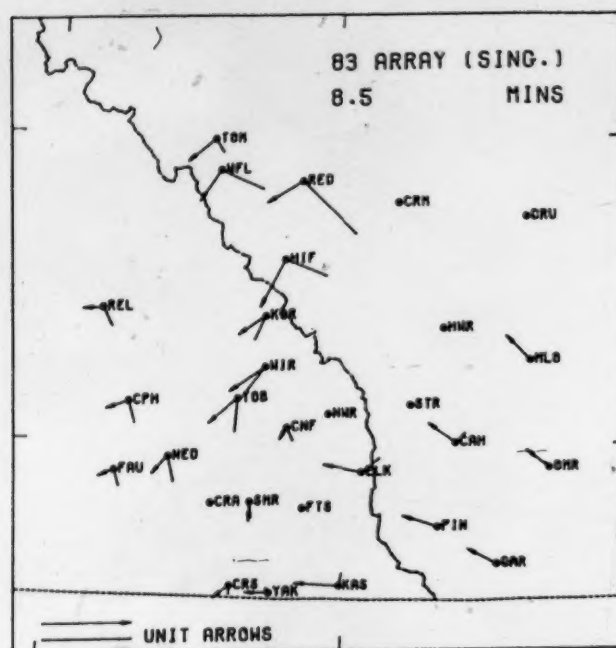


Figure 4.2 Induction arrows from single-station transfer functions (top) and from average-array transfer functions (bottom), at  $T = 8.5$  min.

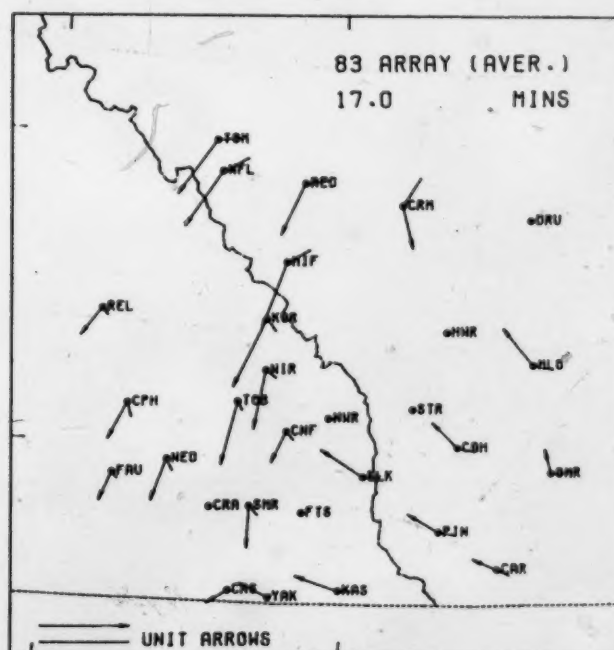
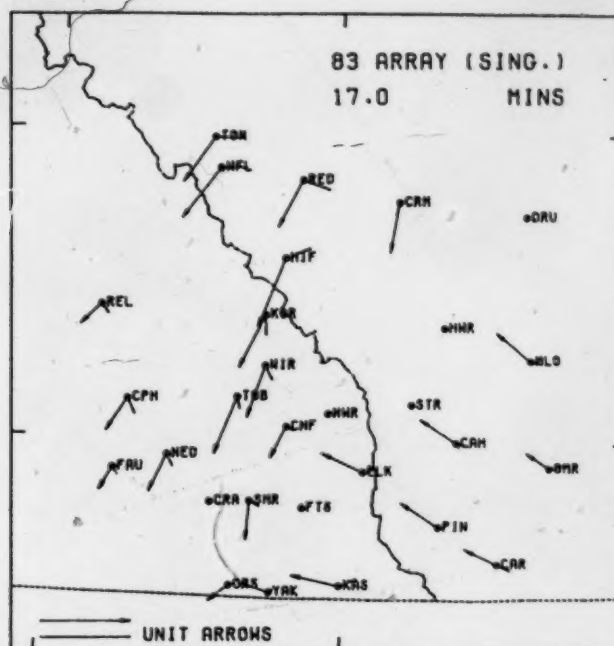


Figure 4.3 Induction arrows from single-station transfer functions (top) and from average-array transfer functions (bottom), at  $T = 17.0$  min.

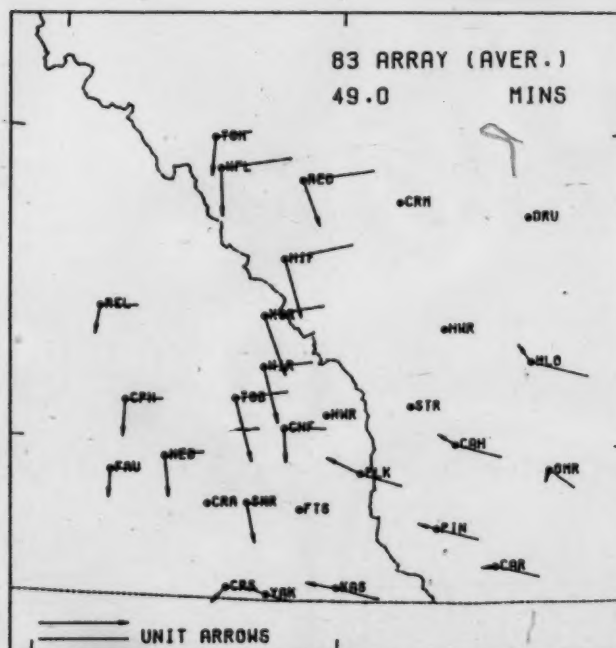
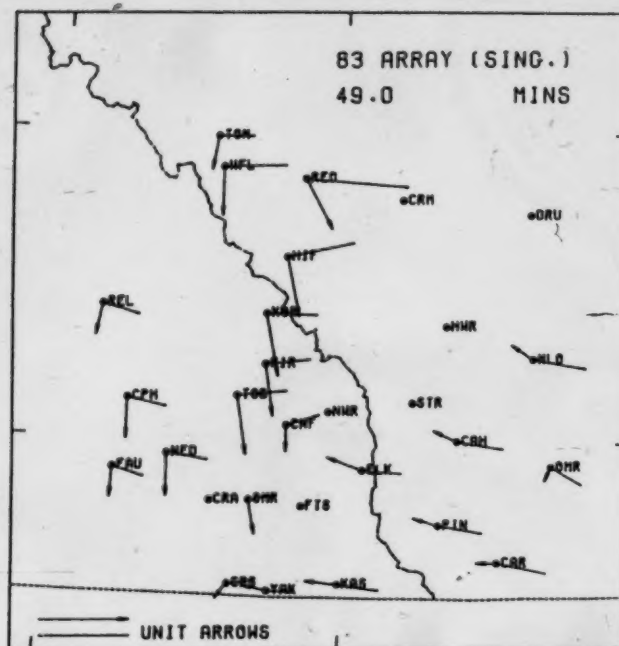


Figure 4.4 Induction arrows from single-station transfer functions (top) and from average-array transfer functions (bottom), at  $T = 49.0$  min.

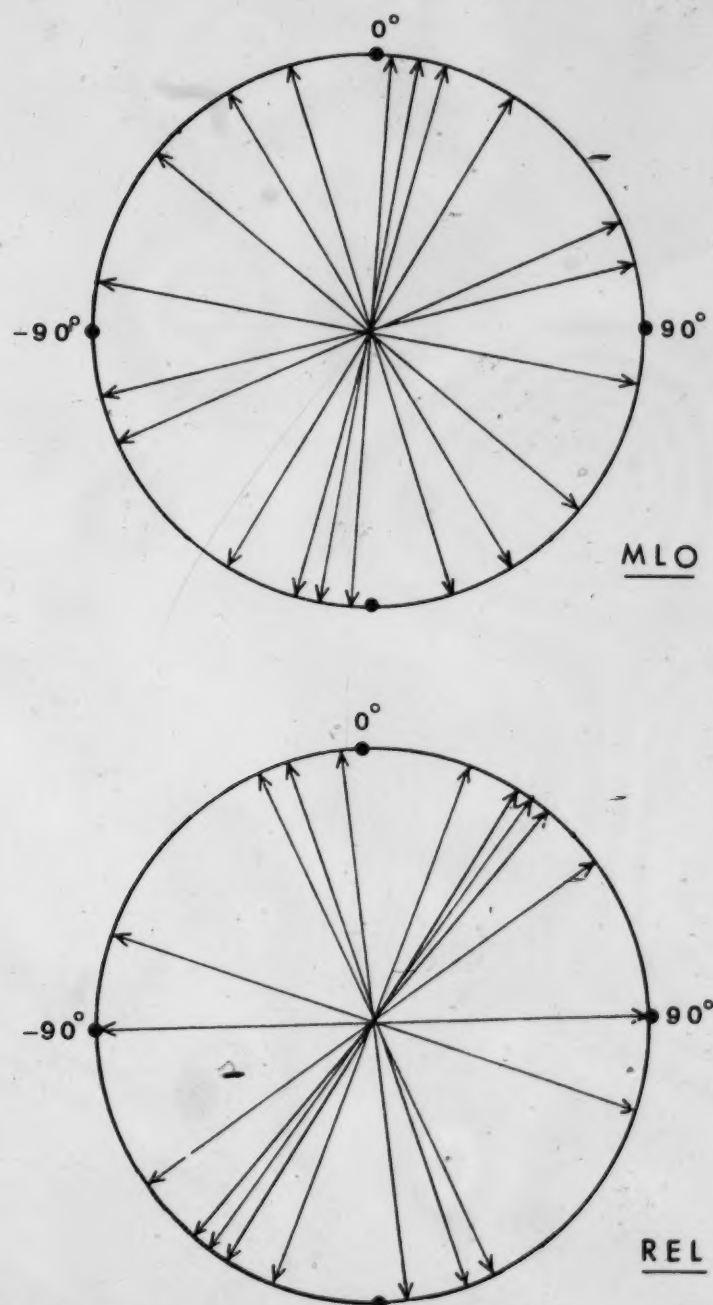


Figure 4.5 Polarization distribution of the events for the period 49.0 min at stations REL and MLO.

Quadrature-phase induction arrows often point toward shallow conductors. The quadrature-phase induction arrows for period 49.0 min are quite consistent over the whole array and point eastward or southeastward. This may be associated with the sedimentary basin in the southern Alberta Plains. The quadrature-phase induction arrows for periods 8.5 and 17.0 min appear to show near surface small scale geological features.

#### 4.2 Average-Array Transfer Functions

The definition of normal horizontal components  $X_n$  and  $Y_n$  in the estimation of transfer functions is a difficult problem. In calculating single-station transfer functions, the observed horizontal components at each station are regarded as normal horizontal components. However, there are various ways of defining the normal horizontal components (Gough and de Beer, 1980). One can take the observed horizontal components at a simultaneously recording normal station far from anomalous conductive structures to represent the normal horizontal components at each station; or a mean value of the simultaneously recording horizontal components over the whole array; or a mean over part of the array away from anomalies. If the mean value of the observed horizontal components over the whole array is taken to represent the normal horizontal components at each station, the transfer functions so estimated are called average-array transfer functions. Average-array transfer function for the



three periods selected for the single-station transfer function were used in calculating the induction arrows shown in the lower halves of Figures 4.2, 4.3 and 4.4.

Comparison of the two sets of induction arrows at three corresponding periods in Figure 4.2, 4.3 and 4.4, shows only slight changes in either magnitude or direction. This indicates that the observed horizontal field is sufficiently uniform over the whole area for the normal field to predominate in the horizontal components.

#### 4.3 Hypothetical Event Analysis

When transfer functions A and B have been estimated over an array, a vertical magnetovariation field Z can be calculated from (4.8) by assuming a uniform normal horizontal field, with any desired polarization. Provided the transfer functions are good estimates from a variety of events with different external current geometries and field polarization, the variations in Z from station to station will be mainly of anomalous origin and will have less normal Z than any single variation event. This method, first used by Bailey et al. (1974), is called hypothetical event analysis. Actually hypothetical event analysis is an alternative method of presenting transfer functions, in place of induction arrows. Examples of its use are given by many others (Bailey and Edwards, 1976; Bailey et al., 1978; Greenhouse and Bailey, 1981; Ingham et al., 1983 and Bingham et al., 1985). Figures 4.6-4.11 give the results of

hypothetical event analyses for transfer functions at periods 8.5, 17.0 and 49.0 min. Uniform horizontal fields, polarized either N45°E (+45° on the maps) or N45°W (-45°) were applied: the former polarization produces maximum induction in conductors elongated NW-SE, the latter polarization in conductors trending NE-SW. Maps marked "Z REAL" correspond to real transfer functions, and show vertical fields of induced currents in phase with the inducing horizontal field; those marked "Z IMAG" show vertical fields of induced currents in quadrature phase with the applied horizontal field.

In all of these maps the conductive structures show with better definition and are more consistent between maps, than the case in the Fourier transform maps of single magnetovariation events shown in Chapter 3. This is because the averaging of several events, in estimating the transfer functions, averages the normal fields and leaves the anomalous field dominant. On the other hand, FT maps of single events have the advantage of showing structures in the horizontal field components as well as the vertical component. Figure 4.6 shows the calculated in-phase vertical field at the period 8.5 min for a linear horizontal field directed along N45°W. The essential feature of this map is that it very clearly shows both of the Southern Alberta-British Columbia (SABC) and Northern Rockies (NR) conductors. The NR conductor shows a positive anomaly at stations WFL and HIF. An anomaly at SMR may not be real, as

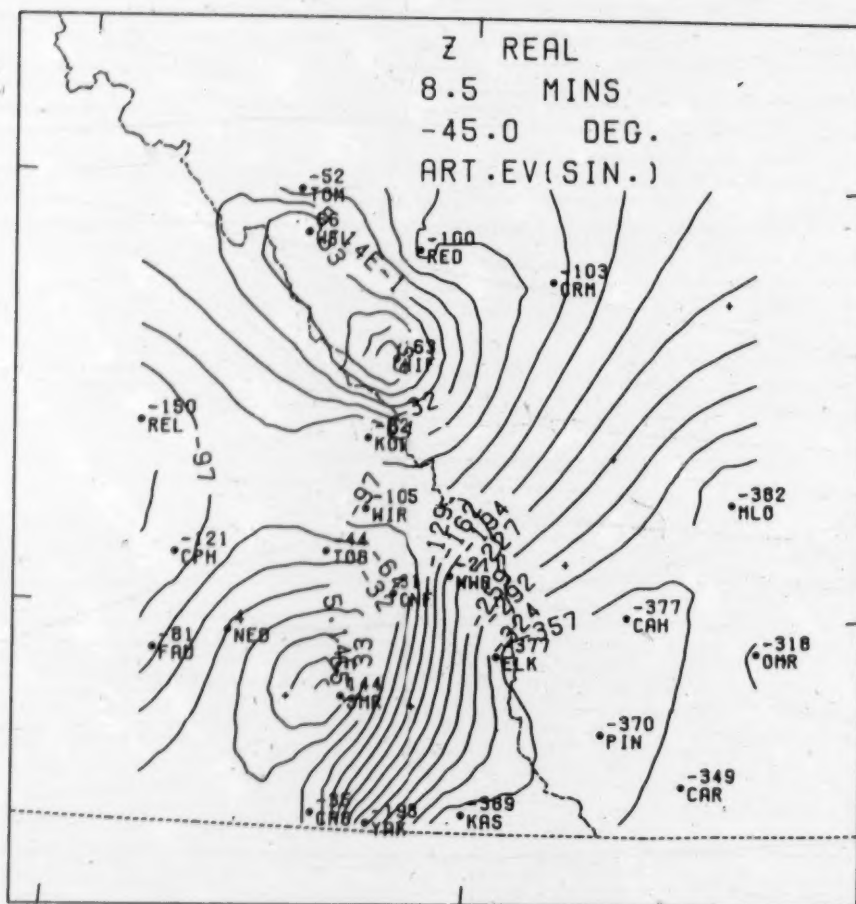


Figure 4.6. Contour map of the in-phase vertical field generated by unit uniform, linearly polarized horizontal magnetic field along  $N45^{\circ}W$  from the single-station transfer functions, at  $T = 8.5$  min.

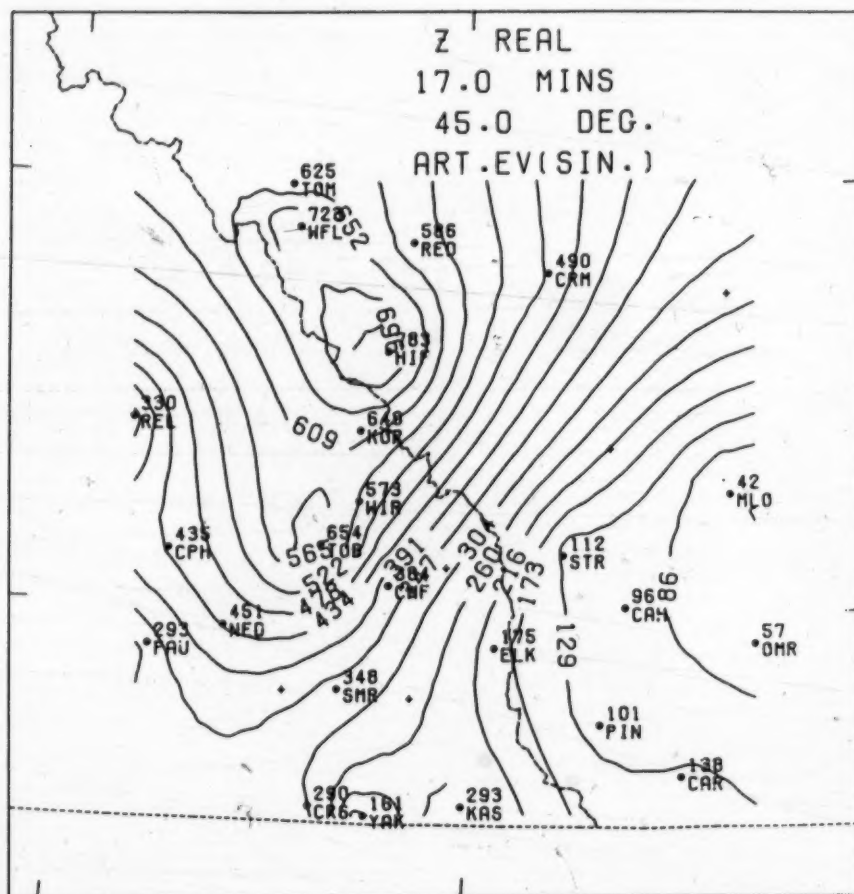


Figure 4.7 Contour map of the in-phase vertical field generated by unit uniform, linearly polarized horizontal magnetic field along N45°E from the single-station transfer functions, at T = 17.0 min.

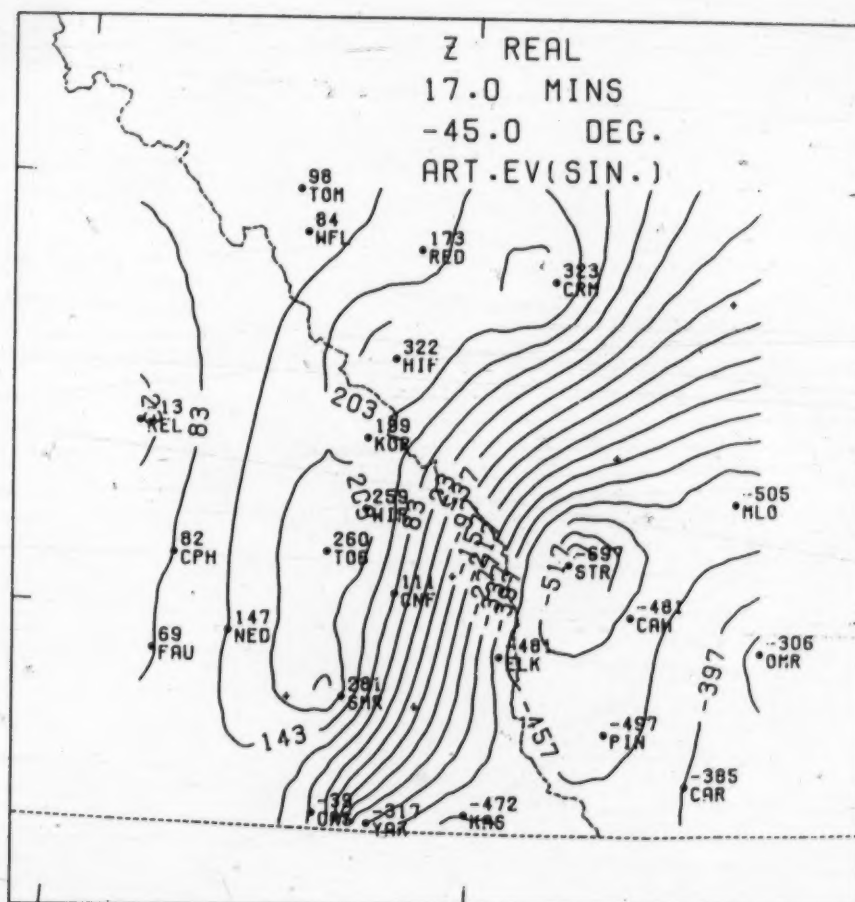


Figure 4.8 Contour map of the in-phase vertical field generated by unit uniform, linearly polarized horizontal magnetic field along  $N45^{\circ}W$  from the single-station transfer functions, at  $T = 17.0$  min.





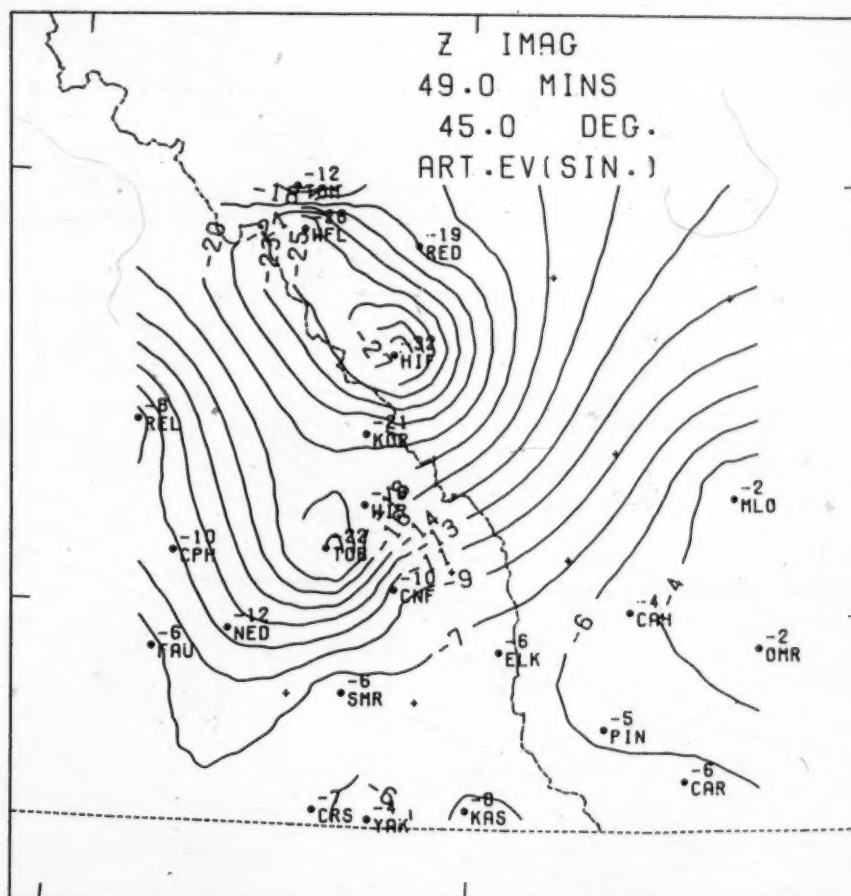


Figure 4.10 Contour map of the quadrature vertical field generated by unit uniform, linearly polarized horizontal magnetic field along  $N45^{\circ}E$  from the single-station transfer functions, at  $T = 49.0$  min.

only one station shows it. If real it is a shallow local feature, as it does not exist in the maps of longer period fields at 17.0 and 49.0 min. Figure 4.7 shows the contour map of the calculated in-phase vertical field at the period 17.0 min for a linear horizontal field directed along  $N45^{\circ}E$ . It clearly shows the NR conductor, and effects of the SABC and CCR conductors; however, in the map for a linear horizontal field directed along  $N45^{\circ}W$  (Figure 4.8), the anomaly of the NR conductor disappears and the map is dominated by the SABC conductor and to a lesser extent the CCR conductor. Similarly Figure 4.9, which is a contour map of calculated in-phase vertical field at the period 49.0 min for a linear horizontal field directed along  $N45^{\circ}W$ , currents in the SABC and CCR conductors are prominent, but the NR conductor is not seen. However, currents in the NR conductor are very well shown in the contour map of the quadrature-phase vertical field for a linear horizontal field directed along  $N45^{\circ}E$  at the period 49.0 min, shown in Figure 4.10. This polarization of the horizontal field produces maximum induction in the NR conductor, which runs NW-SE along the Main Ranges of the Rockies, but does not in the NE-SW trending SABC structure.

It is found that the results of hypothetical event analysis (HEA) from the average-array transfer functions differ very little from HEA results from single-station transfer functions. Figure 4.11 shows an HEA map from average-array transfer functions, for comparison with Figure

4.8. These two maps are almost identical. No further HEA maps from average-array transfer function will be considered

Figures 4.8, 4.11, 4.6 and 4.9 very clearly show the SABC and CCR conductors, whereas Figures 4.6, 4.7 and 4.10 show the NR conductor. The NR conductor has been investigated between  $54^{\circ}$  and  $52^{\circ}\text{N}$  by Bingham et al. (1985). HEA maps in that paper show that it lies under the main Ranges of the Rockies, as delineated by the crowded contours in Figures 4.12 and 4.13. Figures 4.6, 4.7 and 4.10 show an anomaly pattern much like those in Figures 4.12 and 4.13, especially the former. Hence the present investigation gives evidence that the NR conductor continues to the SABC conductor from  $52^{\circ}\text{N}$  along the Main Ranges of the Rockies. Therefore the Northern Rockies conductor runs for at least over 400 km along the Main Ranges of the Rockies from  $54^{\circ}\text{N}$  to  $50.8^{\circ}\text{N}$ . The southern limit of the NR conductor has been located. The northern limit remains for future study. The SABC conductor runs from the Idaho-British Columbia border northeastward across the Rocky Mountains, entering Alberta and passing through Drumhellar. Therefore one can say that the eastern limit of the CCR conductor is formed by the NR conductor north of  $50.8^{\circ}\text{N}$ , and the SABC conductor between  $50.8^{\circ}$  and  $49^{\circ}\text{N}$ .

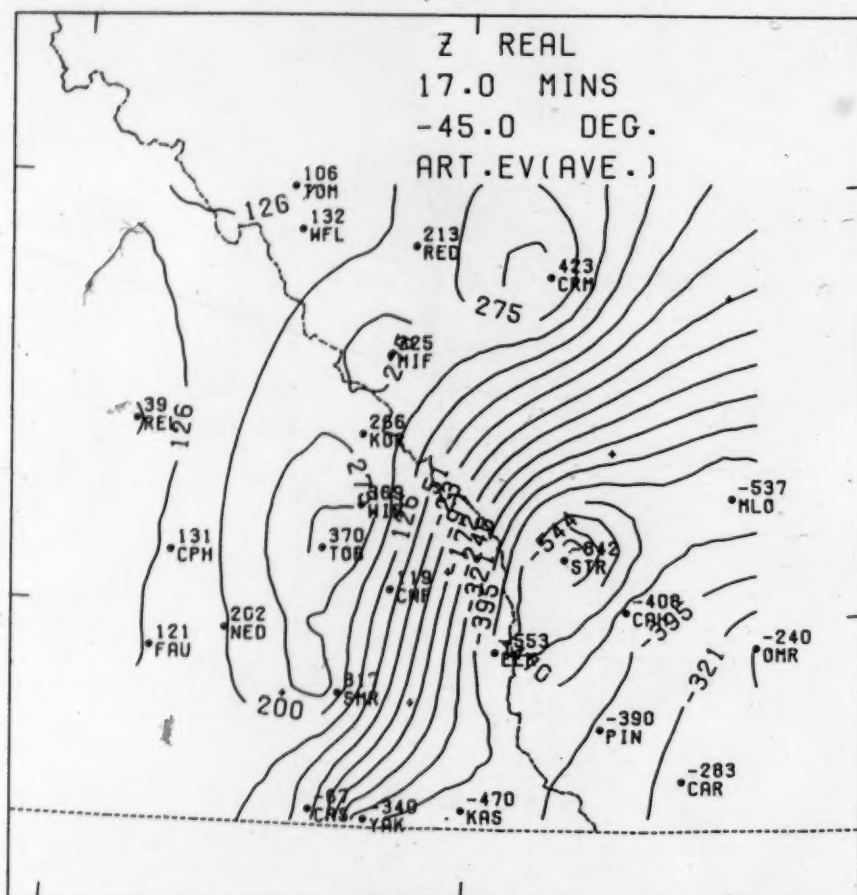


Figure 4.11 Contour map of the in-phase vertical field generated by unit uniform, linearly polarized horizontal magnetic field along N45°W from the average-array transfer functions, at  $T = 17.0$  min.



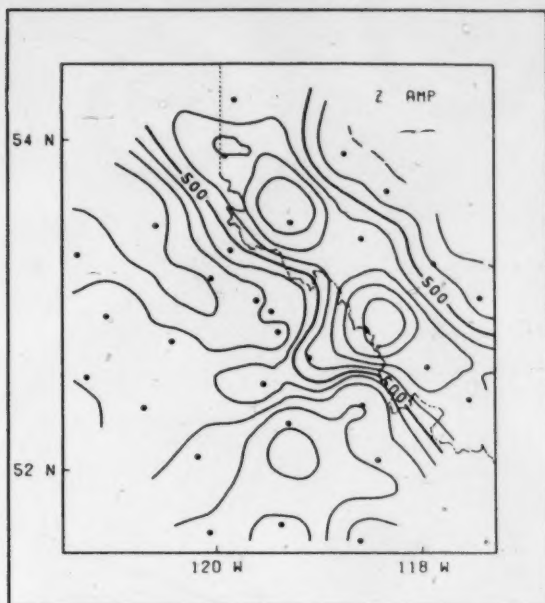


Figure 4.12 Contour map of Z-component field of currents induced by uniform, linearly polarized horizontal magnetic field along N50°E, at T = 10 min. From Bingham et al., 1985.

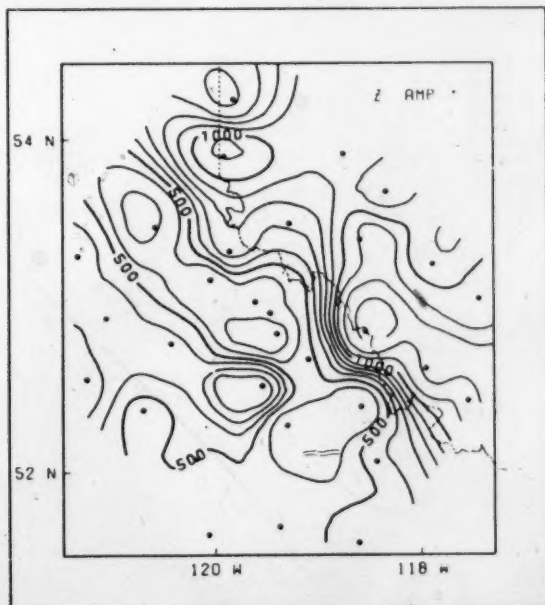


Figure 4.13 Contour map of Z-component field of currents induced by uniform, linearly polarized horizontal magnetic field along N50°E, at T = 22 min. From Bingham et al., 1985.

#### 4.4 Combined Analysis of Magnetovariation Arrays 1981B And 1983

Single-station and average-array transfer functions have been calculated from 10 independent events recorded by array 1981B at periods 10.0 to 19.1 min, and 16 independent events recorded by array 1983 at periods 10.0 to 19.9 min. The resulting induction arrows are shown in Figures 4.14 and 4.15, from single-station transfer functions and from average-array transfer functions respectively. Their average period is 13.9 min. The in-phase arrows in both maps suggest a central axis of the SABC conductor shown by the heavy line. The results are consistent with previous ones and show the SABC conductor crosses Alberta, entering into Saskatchewan. The combined analysis of the two arrays locates the SABC conductor more accurately than either alone. Nevertheless, it must be remembered that fields of external currents always contaminate transfer functions because the simplifying assumptions are only approximately true (Bingham et al., 1985). In Figures 4.14 and 4.15, the stations near the northeastern corner of combined array are near the auroral zone where external currents produce large and variable errors in the transfer functions, as can be seen from the large differences between the inductions — arrows in the NE corners of these two maps. The northeastern end of the line marking the SABC conductor is much less reliable than the remainder.

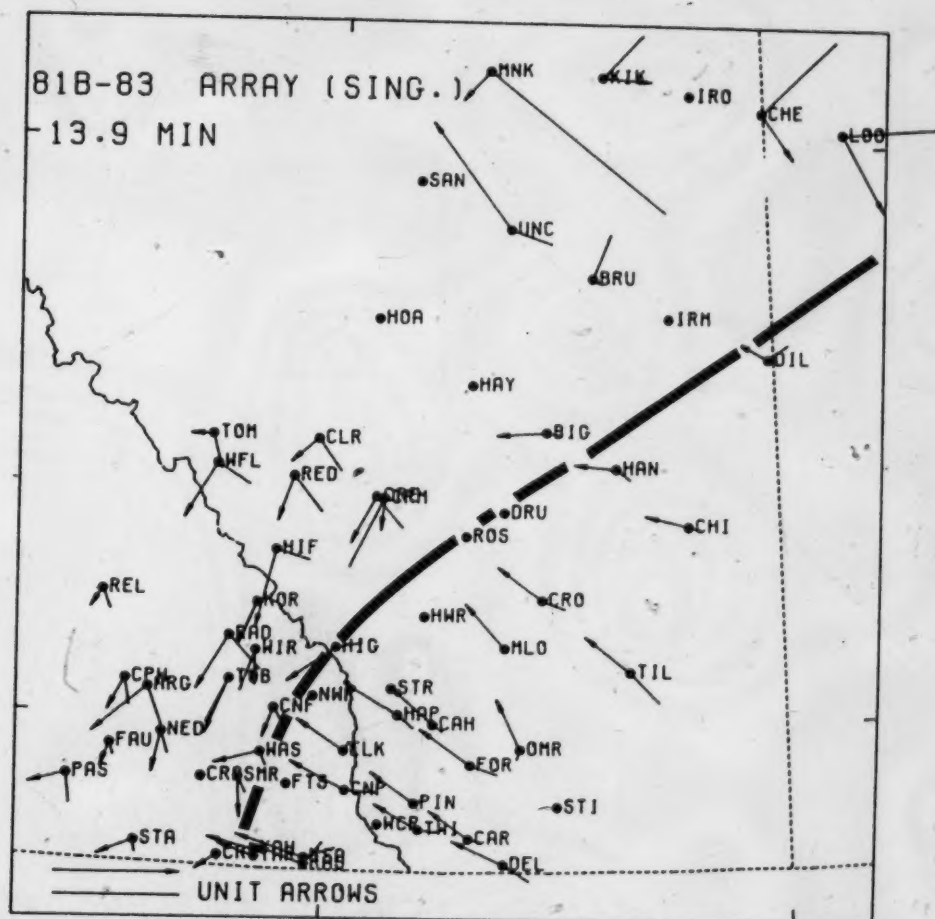


Figure 4.14 Induction arrows from single-station transfer functions of the combination array, near  $T = 13.9$





Figures 4.16 and 4.17 show contour maps of calculated vertical fields from single-station transfer functions of the combined array, for linear horizontal fields directed along  $N45^{\circ}W$  and  $N45^{\circ}E$  respectively. They show the severe contamination by external currents near the northeastern corner, close to auroral electrojet currents. In the southern part of the array Figure 4.16 shows the presence of the SABC conductor, while Figure 4.17 shows the NR conductor. On close examination, however, the values at two close stations from the two arrays, say CPH and ARG, are found to be quite different (283 at CPH and 581 at ARG). Even at the same site, say KSA(1981B) and KAS(1983), the values differ (264 at KSA and 197 at KAS). Clearly the transfer functions even at southern stations are subject to contamination by the external current fields. This contamination produces large errors and complicates the contours in Figures 4.16 and 4.17.

It should be noted that the transfer functions for the stations of the 1983 array are less contaminated by external fields than those of the 1981B array, because more events having varied current geometries and polarization were used for the 1983 array.





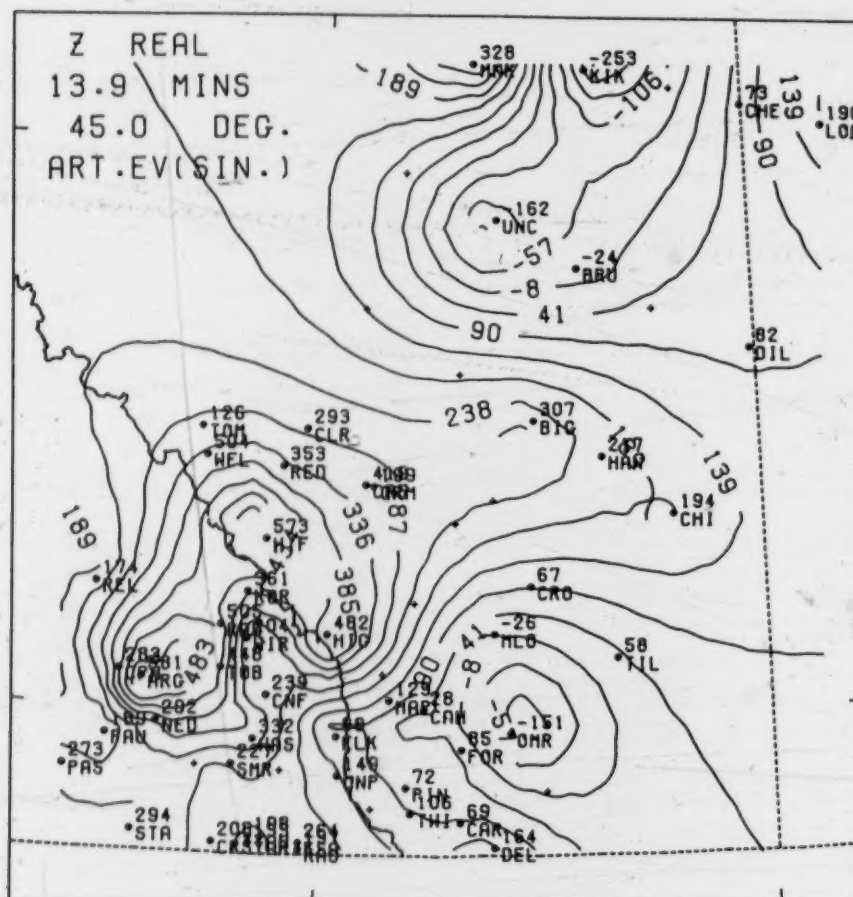


Figure 4.17 Contour map of the in-phase vertical field generated by unit uniform, linearly polarized horizontal magnetic field along N45°E from the single-station transfer functions of combination array, at  $T = 13.9$  min.

## 5. AN APPLICATION OF DOWNWARD CONTINUATION TO MAGNETOVARIATION FIELDS FOR DEPTH ESTIMATION OF THE SABC CONDUCTOR

### 5.1 Downward Continuation of Magnetovariation Field into a Homogeneous Half-Space with a Conductivity $\sigma = 0$

The method of downward continuation appeared in the geophysical literature as early as 1936 (Evjen). Peters (1949) gave a practical scheme of computation. Further papers on this topic were published by Henderson and Zeitz (1949), Roy (1960, 1966a) and others. These developments were aimed mainly at the interpretation of gravity and static magnetic field anomalies. Roy (1966b, 1968 and 1969) developed downward continuation techniques for electromagnetic fields. In the present thesis the method of downward continuation is applied to the interpretation of magnetovariation array data. The use of two-dimensional fast Fourier transforms makes the computation very efficient.

In magnetovariation study the frequencies used are so low that the effects of displacement currents can be neglected. Equation (2.8) is applicable and this is an equation of magnetostatics. We assume that the electrical conductivity of the country rock or enclosing overburden is much smaller than that of the anomalous body. Hence outside the anomalous body equation (2.8) can be written as

$$\nabla \times \vec{H} = 0 \quad (5.1)$$

To a close approximation,  $\vec{H}$  is the gradient of a scalar

potential  $\Phi$ ,

$$\vec{H} = -\nabla\Phi \quad (5.2)$$

where  $\Phi$  satisfies Laplace's equation

$$\nabla^2\Phi = 0 \quad (5.3)$$

Taking the curl of (5.1) with magnetic permeability  $\mu =$  constant,

$$\nabla^2\vec{H} = 0 \quad (5.4)$$

$$\text{or} \quad \nabla^2 H_j = 0 \quad (j=x,y,z) \quad (5.5)$$

Actually, we have obtained equation (5.4) in Chapter 2 (see equation (2.16)).

To take account of phase variations we write

$$H_j = H_{jR} + iH_{jI} \quad (5.6)$$

where the subscripts R and I indicate real and imaginary parts of the component  $H_j$ .

From (5.5) and (5.6) we get

$$\nabla^2 H_{jR} = 0 \quad (5.7)$$

$$\nabla^2 H_{jI} = 0 \quad (5.8)$$

Equation (5.3), (5.4), (5.5), (5.7) and (5.8) have indicated that  $\Phi$ ,  $\vec{H}$ ,  $H_j$ ,  $H_{jR}$  and  $H_{jI}$  individually satisfy Laplace's equation. Therefore, the techniques of downward continuation used for gravity and magnetostatic fields can be applied to any of these quantities.

Following Grant and West (1965, see formulas (8-12) and (11-11)), in the space-frequency domain

$$H_{zR}(x, y, 0, \omega) = \frac{|z|}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \frac{1}{R^3} \right] H_{zR}(\xi, \eta, z, \omega) d\xi d\eta \quad (5.9)$$

where  $x, y$  and  $\xi, \eta$  represent Cartesian coordinates on the plane Earth's surface ( $z = 0$ ) and on a horizontal plane below the Earth's surface ( $z > 0$ ) respectively (the  $z$  axis is directed downwards);  $R = \sqrt{[(x-\xi)^2 + (y-\eta)^2 + z^2]}$  and  $\omega$  is the frequency of magnetovariation fields.  $H_{zR}$  is assumed to be the real part of the  $Z$  component, but the following procedures are completely applicable to other quantities mentioned above. The downward continuation problem is to calculate  $H_{zR}(\xi, \eta, z, \omega)$  from  $H_{zR}(x, y, 0, \omega)$ , which is known. Taking the Fourier transform of both sides of (5.9) with respect to  $x$  and  $y$ , in the wavenumber-frequency domain

$$F_0(k_x, k_y, 0, \omega) = F_z(k_x, k_y, z, \omega) \exp\{[-\sqrt{(k_x^2 + k_y^2)}]z\} \quad (5.10)$$

where  $F_0(k_x, k_y, 0, \omega)$  denotes the Fourier transform of  $H_{zR}(x, y, 0, \omega)$ , and  $F_z(k_x, k_y, z, \omega)$  the Fourier transform of  $H_{zR}(x, y, z, \omega)$ ;  $k_x$  and  $k_y$  are wave numbers. From equation (5.10)

$$F_z(k_x, k_y, z, \omega) = F_0(k_x, k_y, 0, \omega) \exp\{[\sqrt{(k_x^2 + k_y^2)}]z\} \quad (5.11)$$

where the right hand side of (5.11) is calculable.  $H_{zR}(x, y, z, \omega)$  can be found from the inverse Fourier transform of (5.11). That is

$$H_{zR}(x, y, z, \omega) = F_z^{-1}(k_x, k_y, z, \omega) \quad (5.12)$$

For  $z < 0$ , the continuation is upward and for  $z > 0$ , downward.



In downward continuation (5.11) shows that the integral  $\iint |F_z(k_x, k_y, z, \omega)| dk_x dk_y$  is integrable and  $F_z^{-1}$  exists only when  $F_0(k_x, k_y, 0, \omega)$  attenuates with the shorter wave lengths more rapidly than  $\exp\{\sqrt{(k_x^2 + k_y^2)}z\}$  rises. To ensure the necessary attenuation of higher wave-numbers a high-cut mathematical filter can be used. Thus (5.11) can be written as

$$F_z(k_x, k_y, z, \omega) = H_0(k_x, k_y, 0, \omega) \exp\{\sqrt{(k_x^2 + k_y^2)}z - \gamma[k_x^2 + k_y^2]\} \quad (5.13)$$

The appropriate value of  $\gamma$  depends on the depth  $z$ . The continuation method is valid only in free space, where Laplace's equation is satisfied. When a field is continued to the depth of an anomalous body of high conductivity in which induced currents flow, Laplace's equation no longer holds and the continuation process fails. The continued field then becomes unstable and even oscillatory. This property of the continued fields may be used to estimate the depth of the top of a highly conductive structure and the shape of that structure.

The total magnetic field  $H_t$  observed by the array can be decomposed in two ways:

$$\vec{H}_t = \vec{H}_e + \vec{H}_i \quad (5.14)$$

$$\text{and} \quad \vec{H}_t = \vec{H}_n + \vec{H}_a \quad (5.15)$$

where  $\vec{H}_e$  = external field due to currents above the Earth's surface

$\vec{H}_i$  = internal field due to currents within the  
Earth

$\vec{H}_n$  = normal field

$\vec{H}_a$  = anomalous field

and the normal field may be defined by

$$\vec{H}_n = -\vec{H}_e + \vec{H}_{in} \quad (5.16)$$

where  $\vec{H}_{in}$  is the field of currents induced in the regional structures (one dimensional structures). for our purpose,  $\vec{H}_e$ ,  $\vec{H}_i$  and  $\vec{H}_a$  can be used, but I think that  $\vec{H}_e$  is good;  $\vec{H}_i$  better and  $\vec{H}_a$  best.  $\vec{H}_i$  is used in the present thesis. Therefore, before the continuation is done, we have to separate the internal fields from the total fields.

Note that our observed data are the magnetic induction rather than the magnetic field. It is easy to obtain the formulas for the magnetic induction by using relation (2.6).

## 5.2 The Separation of the Total Magnetovariation Field into External and Internal Parts

The separation of a magnetovariation field observed over a finite array, into external and internal parts, can be considered in several ways. The oldest of these is due to Vestine (1941) and is based upon the application of Green's second identity to magnetic potentials; similar methods were used by Siebert and Kertz (1957), Price and Wilkins (1963) and Hartman (1963). Weaver (1963) used the convolution theorem of two-dimensional Fourier analysis to obtain the

same result. These relations are:

$$2\pi(Z_e - Z_i)|_o = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(x-x_o)X(x,y) + (y-y_o)Y(x,y)}{[(x-x_o)^2 + (y-y_o)^2]^{3/2}} dx dy \quad (5.17)$$

$$2\pi(Y_e - Y_i)|_o = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(y-y_o)Z(x,y)}{[(x-x_o)^2 + (y-y_o)^2]^{3/2}} dx dy \quad (5.18)$$

$$2\pi(X_e - X_i)|_o = - \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(x-x_o)Z(x,y)}{[(x-x_o)^2 + (y-y_o)^2]^{3/2}} dx dy \quad (5.19)$$

The point at which separation is made is  $(x_o, y_o)$ . The observed vertical component is  $Z = Z_e + Z_i$ . Together with  $(Z_e - Z_i)$ , we get  $Z_e$  and  $Z_i$ , and similarly for the other components. The relations (5.17)-(5.19) have singularities at the point  $(x_o, y_o)$ . The singularities can be avoided by considering the contribution of a small square of side  $2a$  centered at the separation point which are (Oldenburg, 1969)

$$2a \ln \left[ \frac{\sqrt{2+1}}{\sqrt{2-1}} \right] \left[ \left. \frac{\partial X}{\partial x} \right|_o + \left. \frac{\partial Y}{\partial y} \right|_o \right] \text{ to } 2\pi(Z_e - Z_i)|_o \quad (5.20)$$

$$- 2a \ln \left[ \frac{\sqrt{2+1}}{\sqrt{2-1}} \right] \left[ \left. \frac{\partial Z}{\partial y} \right|_o \right] \text{ to } 2\pi(Y_e - Y_i)|_o \quad (5.21)$$

$$- 2a \ln \left[ \frac{\sqrt{2+1}}{\sqrt{2-1}} \right] \left[ \left. \frac{\partial Z}{\partial x} \right|_o \right] \text{ to } 2\pi(X_e - X_i)|_o \quad (5.22)$$

These relations have been used to separate the observed fields over a magnetovariation array in both time domain and frequency domain (Oldenburg, 1969; Porath et al., 1970).

Recently Wang et al. (1986) have given a method of separation of magnetovariation fields into external and internal parts in the wavenumber domain. In this thesis the method derived by Wang et al. is used. This method has considerable advantages in speed of computation. Their

derivation is summarized here.

Suppose that a general solution of equation (5.3) in the space domain is, (see (2.35))

$$\Phi = A(x,y)\exp[+\nu z] + B(x,y)\exp[-\nu z] \quad (5.23)$$

$$\text{where } \nu = \sqrt{K_x^2 + K_y^2} \quad (5.24)$$

Taking the Fourier transform of both sides of (5.23) with respect to  $x$  and  $y$ , we get the solution of (5.3) in the wavenumber domain,

$$\begin{aligned} \Phi(\vec{K}, z) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi(x, y, z) \exp[i(k_x x + k_y y)] dx dy \\ &= A(\vec{K}) \exp[+\nu z] + B(\vec{K}) \exp[-\nu z] \end{aligned} \quad (5.25)$$

where

$$A(\vec{K}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(x, y) \exp[i(k_x x + k_y y)] dx dy \quad (5.26)$$

$$\text{and } B(\vec{K}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} B(x, y) \exp[i(k_x x + k_y y)] dx dy \quad (5.27)$$

which describe the terms for internal and external sources, respectively. The magnetic field in  $\vec{K}$  space can be written as

$$\vec{H}(\vec{K}, z) = A(\vec{K}) \vec{R}_a + B(\vec{K}) \vec{R}_b \quad (5.28)$$

where

$$\vec{R}_a = \exp[+\nu z] \begin{bmatrix} i k_x \\ i k_y \\ -\nu \end{bmatrix} \quad (5.29)$$

$$\vec{R}_b = \exp[-\nu z] \begin{bmatrix} i k_x \\ i k_y \\ +\nu \end{bmatrix} \quad (5.30)$$

$$\text{and } \vec{R}_a^\dagger \cdot \vec{R}_b = 0 \quad (5.31)$$

where  $\dagger$  denotes the Hermitean adjoint of a vector.

Equation (5.28) can be written

$$\vec{H}(\vec{R}, z) = A(\vec{R})\vec{R}_a + B(\vec{R})\vec{R}_b + \vec{Q} \quad (5.32)$$

where  $\vec{Q}$  is an error term. Minimizing  $P = \Sigma \vec{Q}^\dagger \cdot \vec{Q}$  with setting  $\partial P / \partial A^*(\vec{R}) = \partial P / \partial B^*(\vec{R}) = 0$  at  $z=0$ , where  $*$  denotes complex conjugate, and using (5.31),  $A(\vec{R})$  and  $B(\vec{R})$  are obtained in terms of observed magnetovariation field components,

$$A(\vec{R}) = \frac{\vec{R}_a^\dagger \cdot \vec{H}(\vec{R})}{\vec{R}_a^\dagger \cdot \vec{R}_a} \quad (5.33)$$

$$B(\vec{R}) = \frac{\vec{R}_b^\dagger \cdot \vec{H}(\vec{R})}{\vec{R}_b^\dagger \cdot \vec{R}_b} \quad (5.34)$$

Substituting (5.33) and (5.34) into (5.28), we obtain the internal and the external parts  $\vec{H}_i$  and  $\vec{H}_e$  at the Earth's surface in the wavenumber domain, respectively,

$$\vec{H}_i(\vec{R}) = \frac{[\vec{R}_a^\dagger \cdot \vec{H}(\vec{R})]\vec{R}_a}{\vec{R}_a^\dagger \cdot \vec{R}_a} \quad (5.35)$$

$$\vec{H}_e(\vec{R}) = \frac{[\vec{R}_b^\dagger \cdot \vec{H}(\vec{R})]\vec{R}_b}{\vec{R}_b^\dagger \cdot \vec{R}_b} \quad (5.36)$$

If we introduce

$$\vec{R}_i = \vec{R}_a / \sqrt{(2\nu \times \nu)} \quad (5.37)$$

$$\text{and} \quad \vec{R}_e = \vec{R}_b / \sqrt{(2\nu \times \nu)} \quad (5.38)$$

$$\text{we get} \quad \vec{H}_i(\vec{R}) = [\vec{R}_i^\dagger \cdot \vec{H}(\vec{R})]\vec{R}_i \quad (5.39)$$

$$\vec{H}_e(\vec{R}) = [\vec{R}_e^\dagger \cdot \vec{H}(\vec{R})]\vec{R}_e \quad (5.40)$$

$$\text{and} \quad \vec{R}_i^\dagger \cdot \vec{R}_i = 1 \quad (5.41)$$

$$\vec{R}_e^\dagger \cdot \vec{R}_e = 1 \quad (5.42)$$

The inverse Fourier transforms of  $\vec{H}_i(\vec{R})$  and  $\vec{H}_e(\vec{R})$  give the internal and external fields in the space domain at the Earth's surface.



A magnetovariation event at period 85.5 mins during 22:20-24:20 UT on October 7, 1981 recorded by the 1981B array was chosen, because the 1981B array covered more of the SABC conductor than the 1983 array. The Z component is considered since  $Z_a$  is a larger fraction of  $Z_t$  than  $X_a$  and  $Y_a$  are of  $X_t$  and  $Y_t$  (see Chapter 2).

Maps of the real and imaginary parts of Fourier transforms of the Z component are shown in Figure 5.1 and 5.2. The maps show the observed total Z component field at the Earth's surface. Some contamination of  $Z_a$  by the external field is obvious, specially at the northeastern corner of the array, which is close to the auroral zone.

The internal parts, separated by the method of Wang et al. (1986), are shown in Figures 5.3 and 5.4. The maps of internal fields show a clear pattern of a current flowing in the SABC conductor. The central contour of parallel contours from the southwest to the northeast probably marks the central axis of the SABC conductor, in agreement with the results inferred from induction arrows, FT anomaly maps and artificial event analyses (Chapter 3-4).

Results of the downward continuation of the internal fields shown in Figures 5.3 and 5.4, to different depths in terms of the methods discussed above are shown in figures 5.5-5.16.



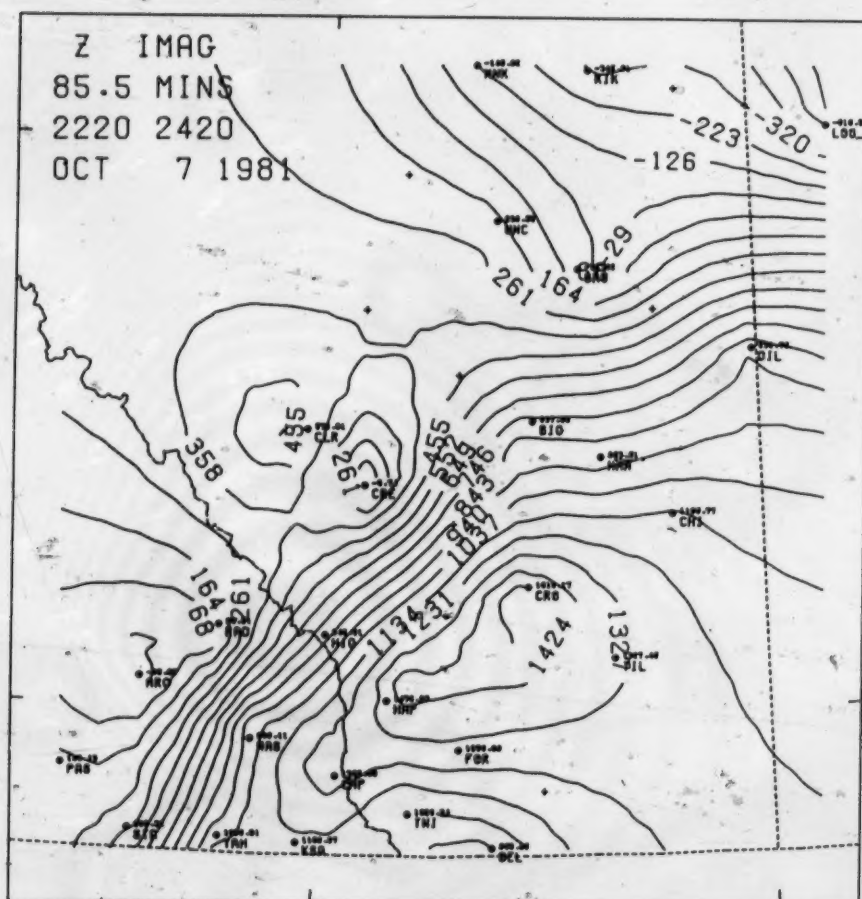


Figure 5.2 Map of FT of imaginary part of a magnetovariation event at period 85.5 minutes during 22:20-24:20 UT on October 7, 1981, recorded by the 1981B array.

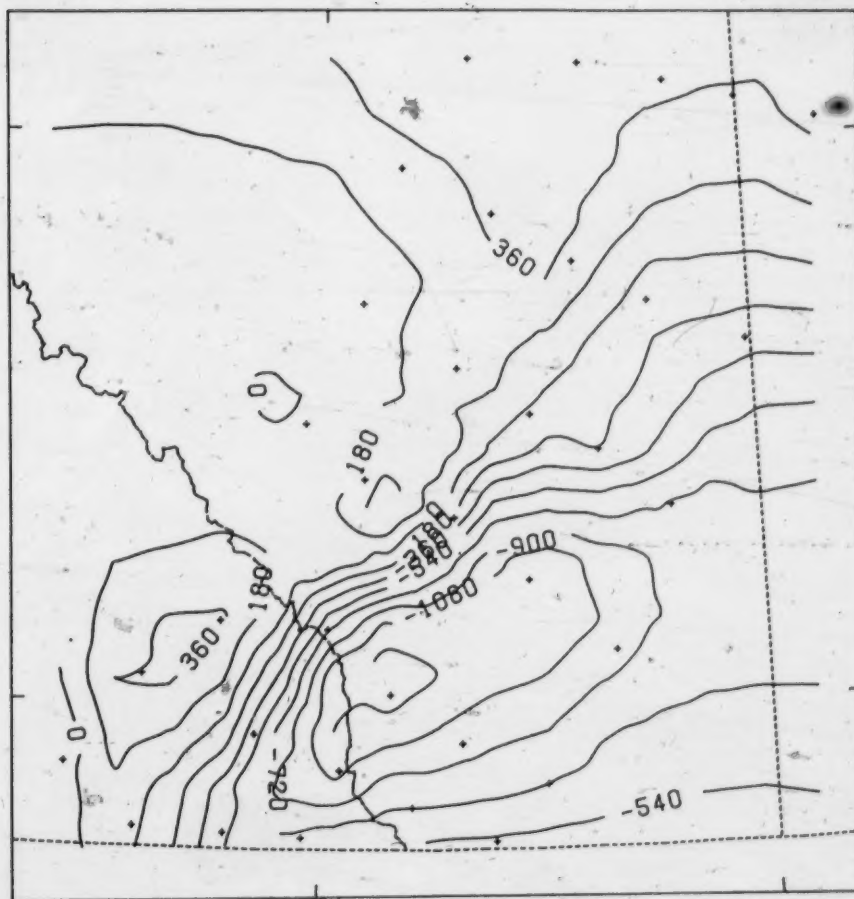


Figure 5.3 Component of the separated internal field at the Earth's surface for the event showing in figure 5.1, real  $Z$  component.

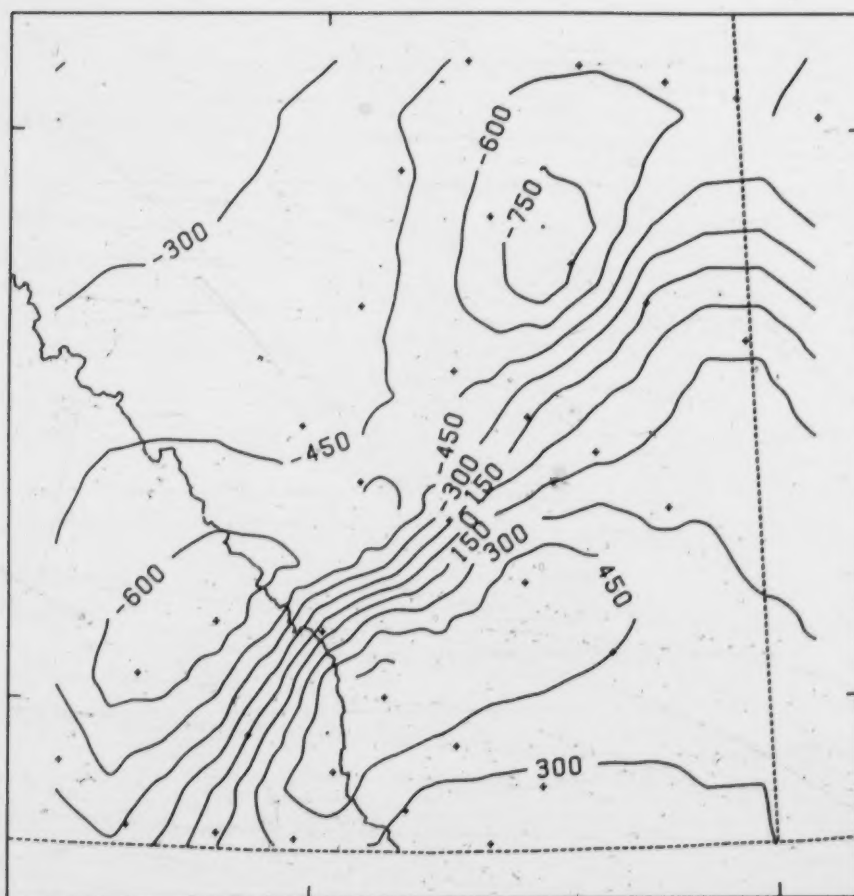


Figure 5.4 Component of the separated internal field at the Earth's surface for the event showing in figure 5.2, imaginary Z component.



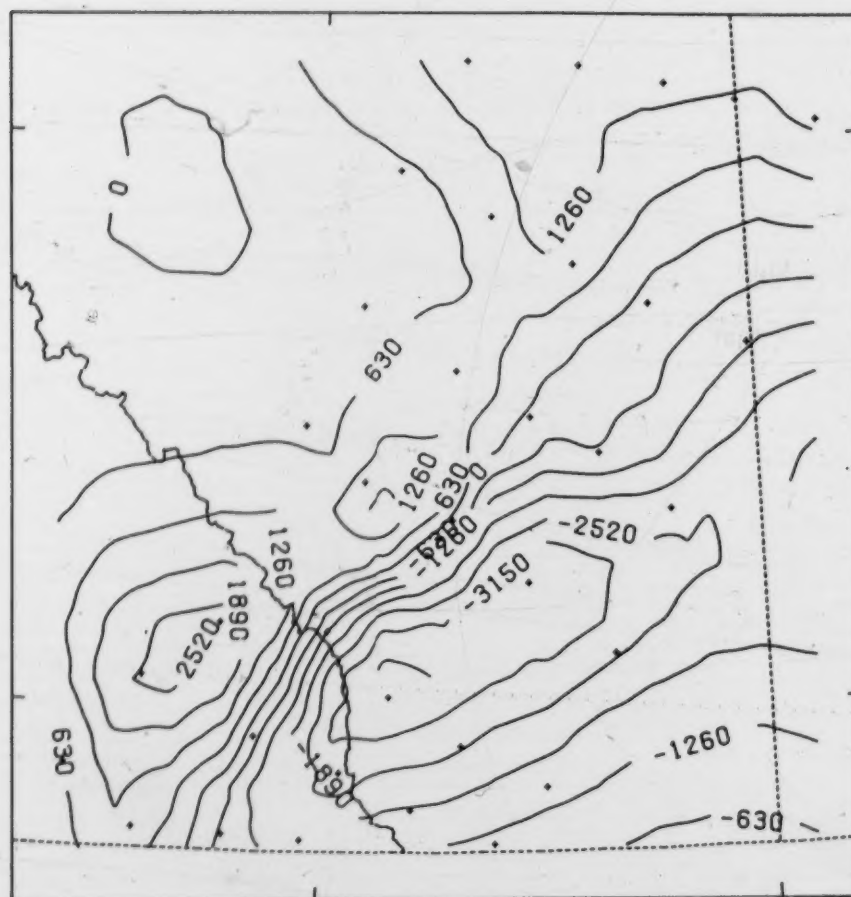


Figure 5.5 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : real  $Z$  component and depth 10 km

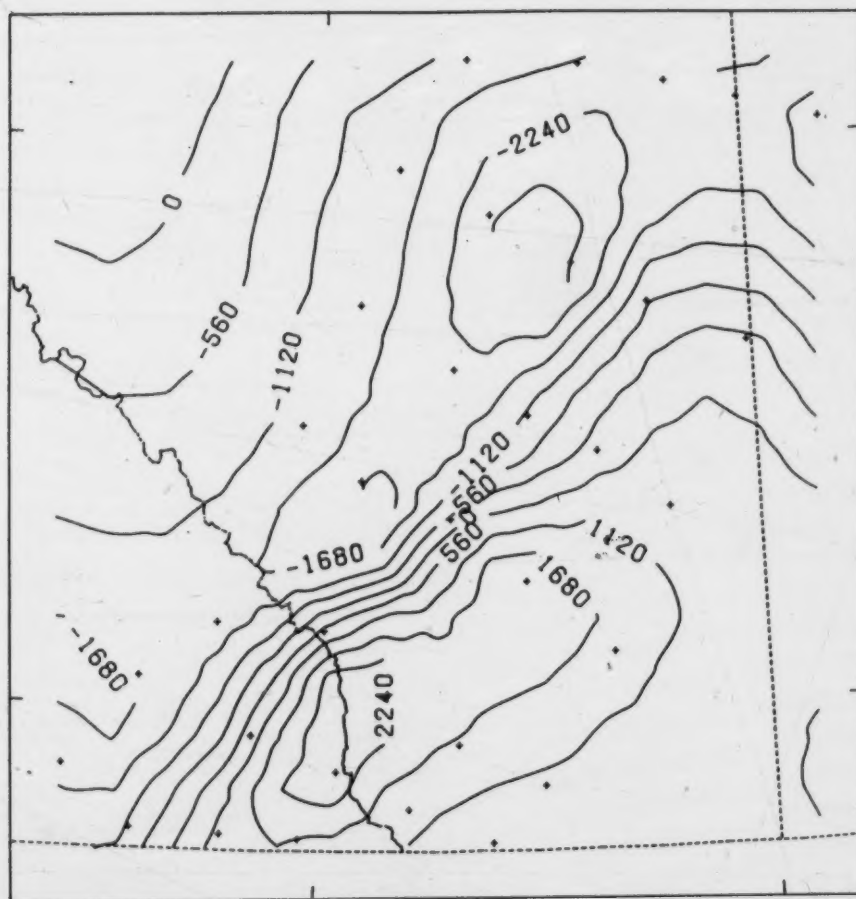


Figure 5.6 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : imaginary Z component and depth 10 km

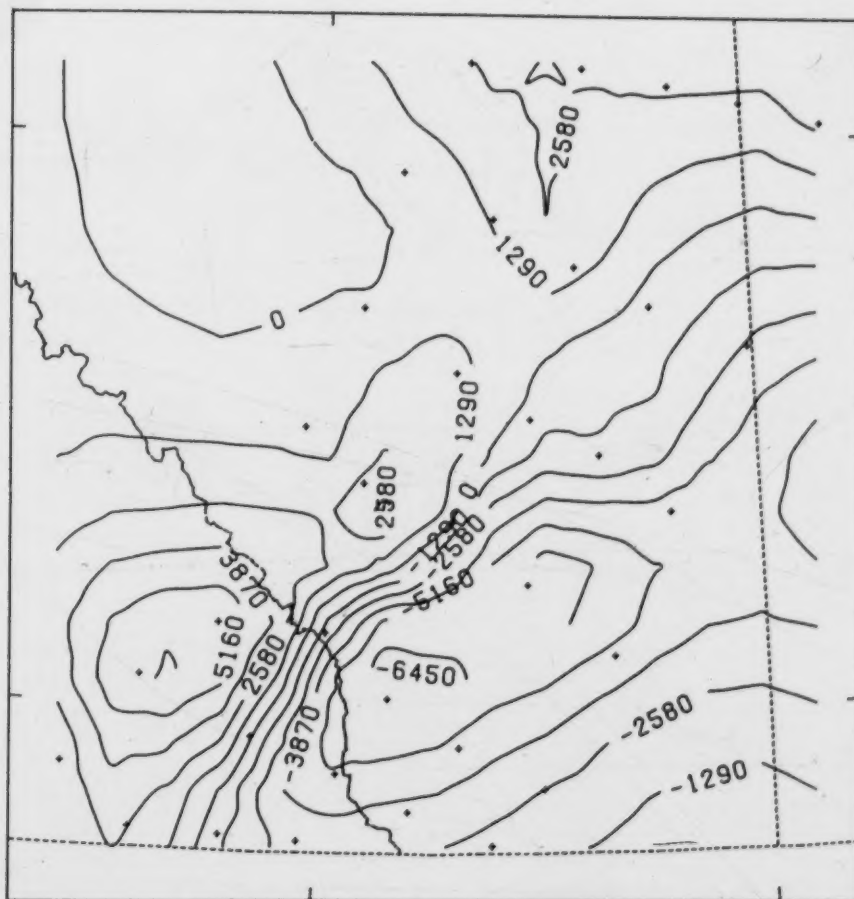


Figure 5.7 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : real  $Z$  component and depth 15 km

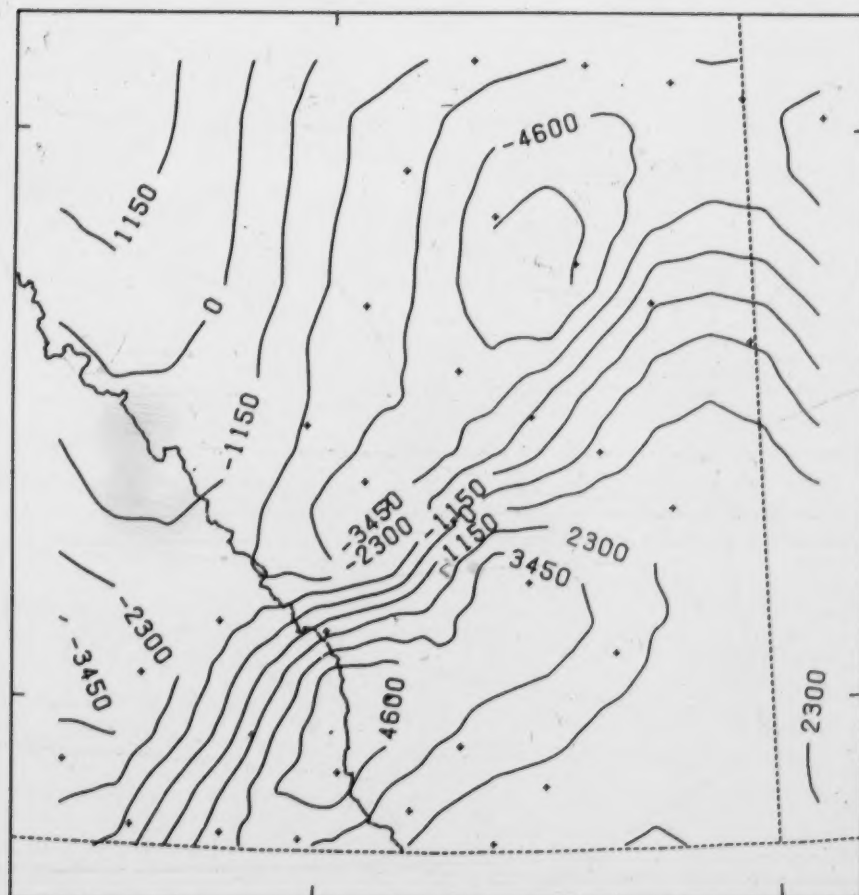


Figure 5.8 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : imaginary  $Z$  component and depth 15 km

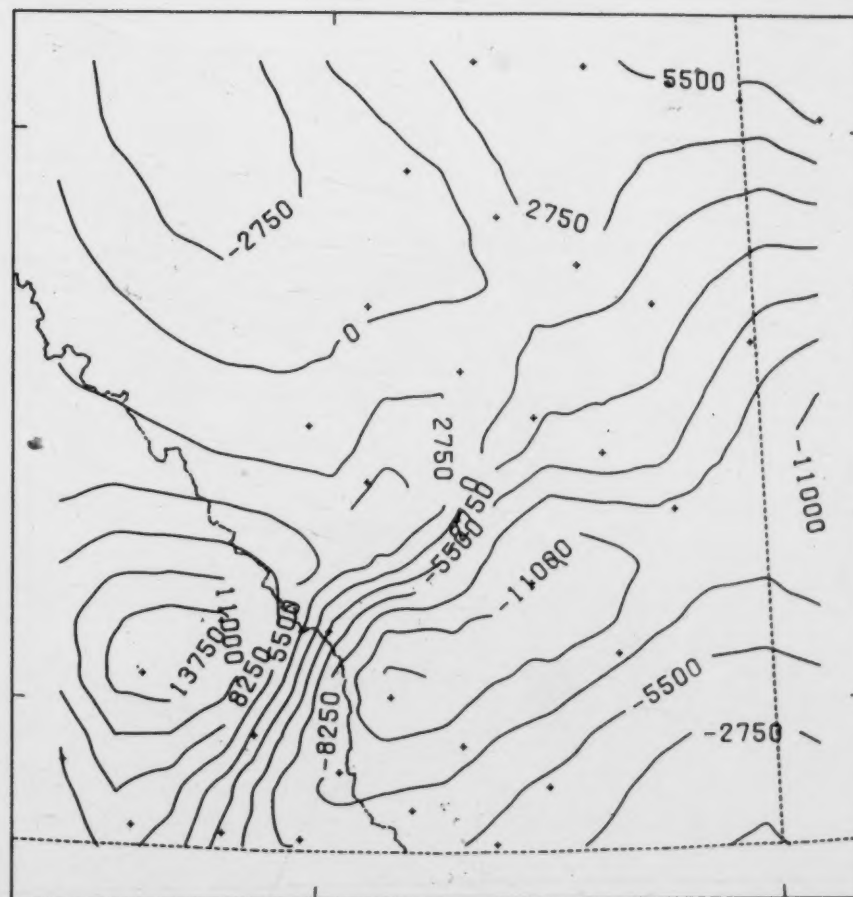


Figure 5.9 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : real  $Z$  component and depth 20 km



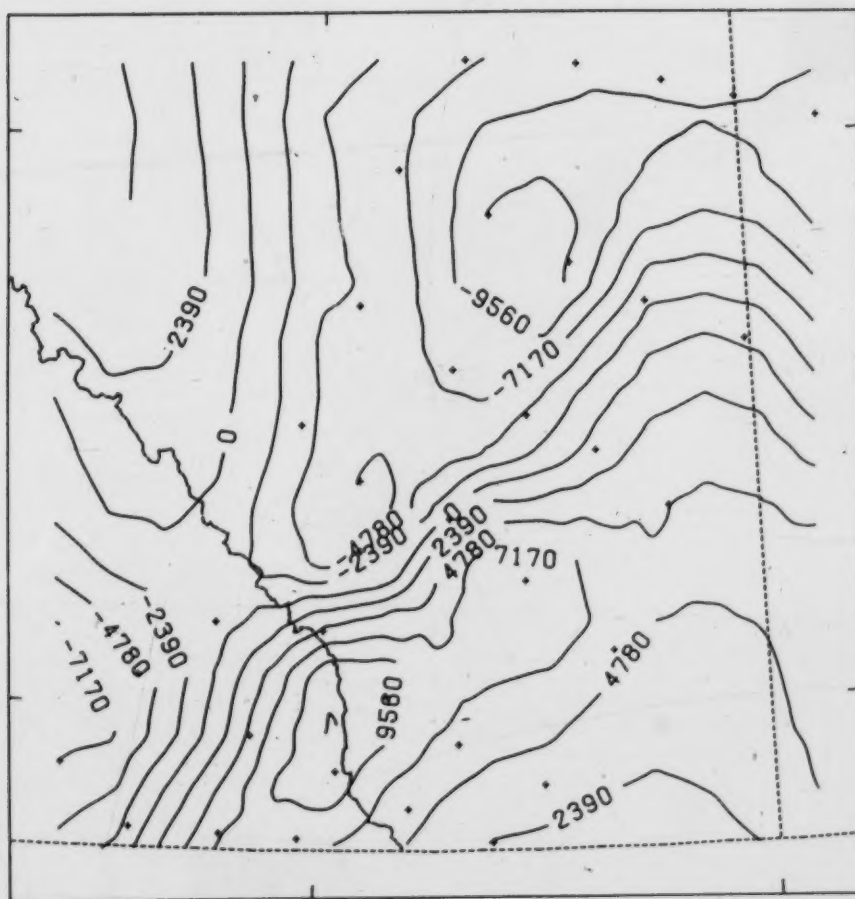


Figure 5.10 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : imaginary Z component and depth 20 km

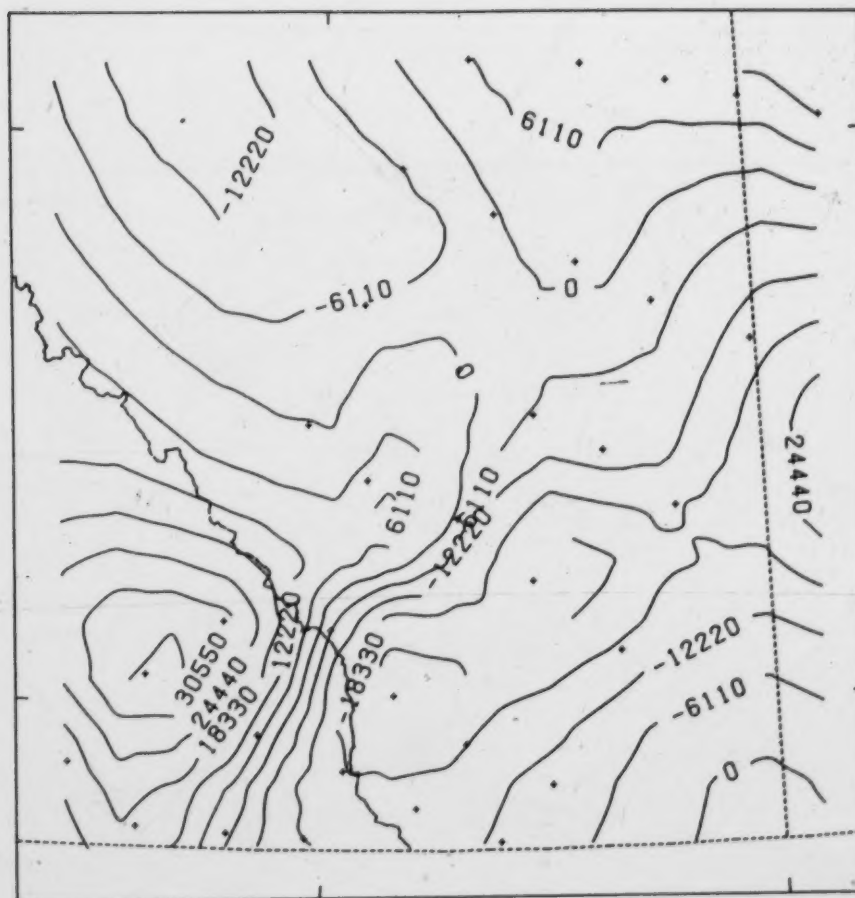


Figure 5.11 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : real  $Z$  component and depth 25 km

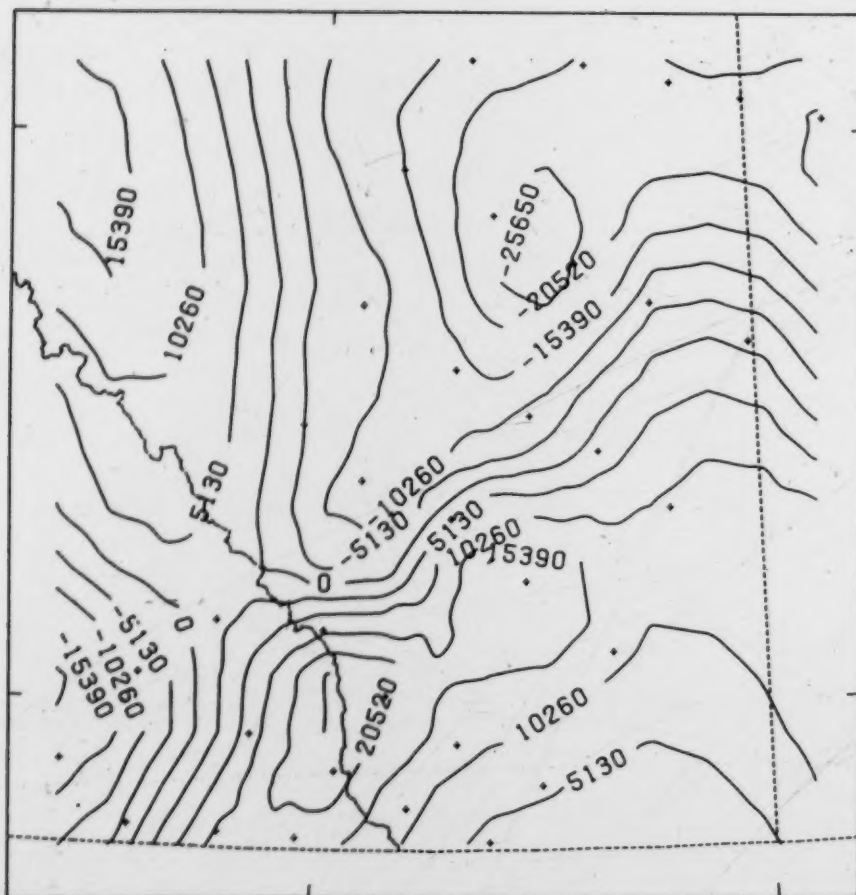


Figure 5.12 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : imaginary Z component and depth 25 km

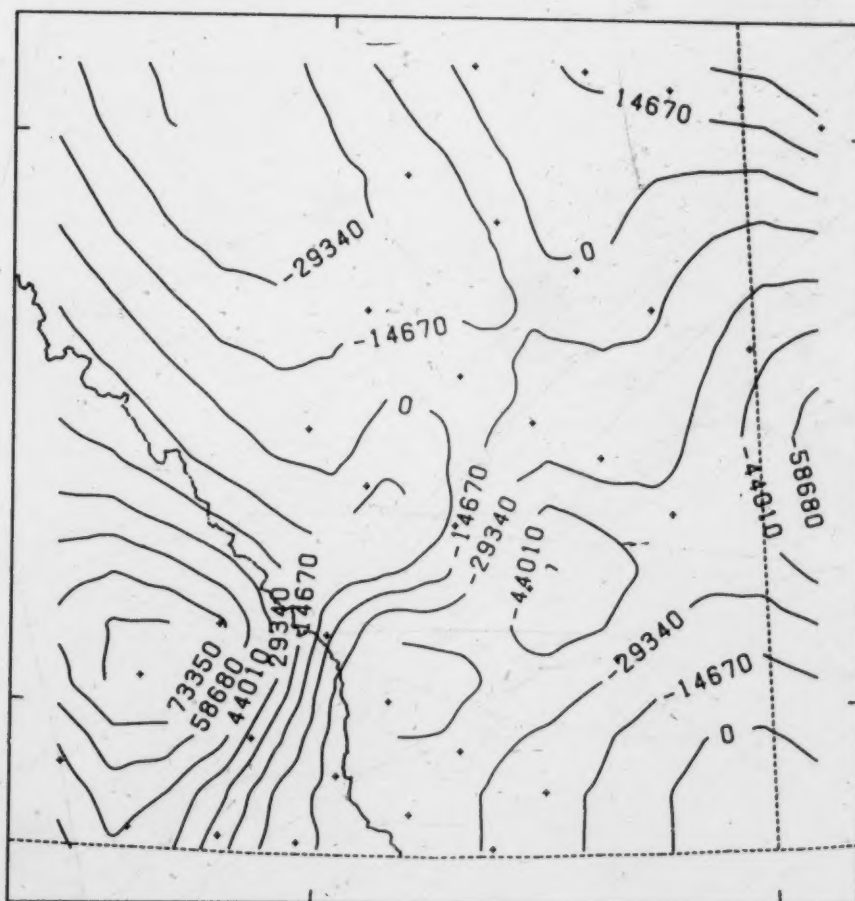


Figure 5.13 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : real Z component and depth 30 km

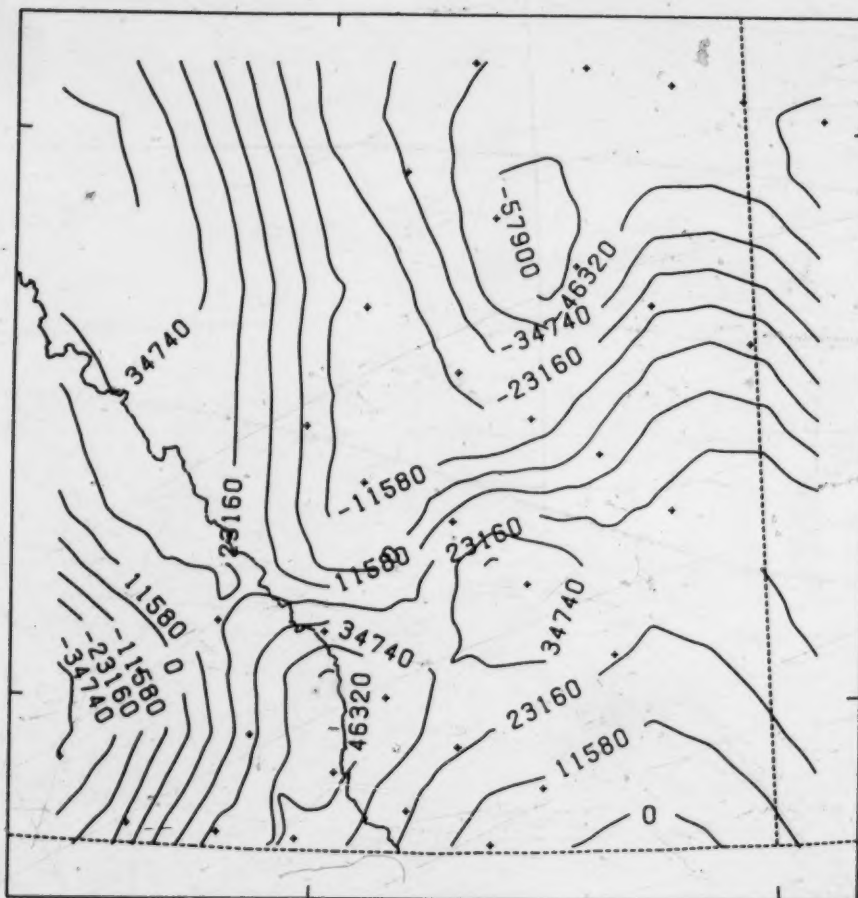


Figure 5.14 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : imaginary Z component and depth 30 km



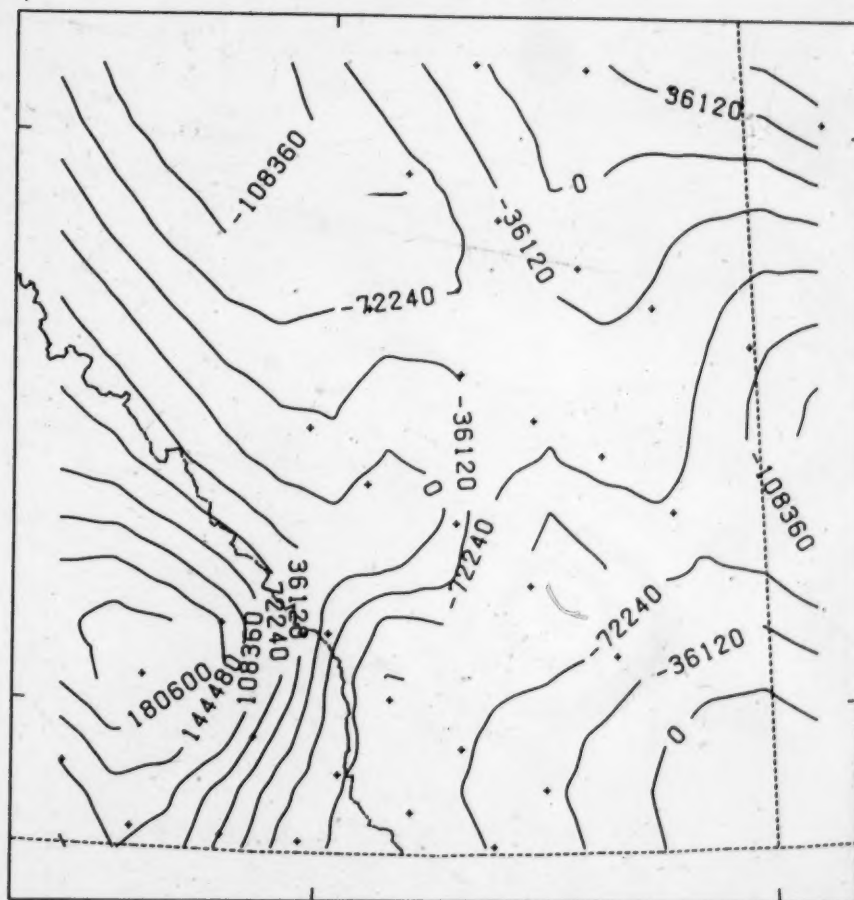


Figure 5.15 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : real  $Z$  component and depth 35 km

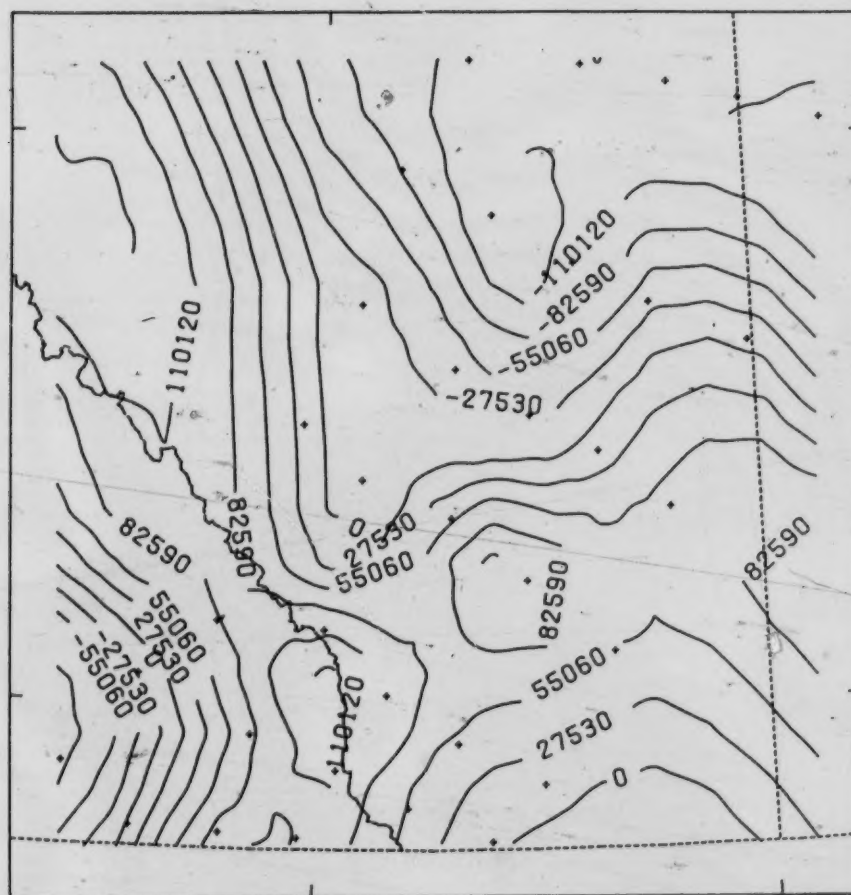


Figure 5.16 Downward continuation of the internal field into a homogeneous half-space with conductivity  $\sigma = 0$ : imaginary  $Z$  component and depth 35 km

### 5.3 Downward Continuation of Magnetovariation Field into a Homogeneous Half-Space with a Finite Conductivity $\sigma_n \neq 0$

When we take account of the Earth's conductivity, the electromagnetic fields satisfy (2.15) instead of Laplace's equations (2.16). (2.15) can be rewritten as

$$\nabla^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} - k_n^2 \begin{bmatrix} \vec{E} \\ \vec{H} \end{bmatrix} = 0 \quad (5.43)$$

where  $\vec{E}$  and  $\vec{H}$  have been replaced by  $\vec{E}$  and  $\vec{H}$  and

$$k_n = \sqrt{i\omega\mu\sigma_n} = (1 + i)\sqrt{\omega\mu\sigma_n/2} \quad (5.44)$$

The value of  $k_n$  is chosen to have a positive real part.

(5.43) is called the Helmholtz equation. The theory of functions of a complex variable states if two functions  $f_1$  and  $f_2$  are analytic respectively in the adjacent domains  $D_1$  and  $D_2$ , having a common boundary  $C$ , then a necessary and sufficient condition that each of these functions is an analytic continuation of the other is that they have equal values on  $C$  (Townsend, 1915; Morse and Feshbach, 1953).

Berdichevsky and Zhdanov (1984) have shown that for an electromagnetic field,  $\vec{E}$  and  $\vec{H}$  need have only equal tangential components on  $C$ . These authors have shown, further, that the functions  $\vec{E}$  and  $\vec{H}$  describing electromagnetic fields in a homogeneous medium are analytic.

The boundary conditions for the electromagnetic fields  $\vec{E}$  and  $\vec{H}$  at an interface between two media are (see Jackson, 1975),

$$(\vec{E}_2 - \vec{E}_1) \times \vec{n} = 0 \quad (5.45)$$

$$(\vec{H}_2 - \vec{H}_1) \times \vec{n} = - \vec{K} \quad (5.46)$$

where  $\vec{n}$  is a unit vector normal to the surface and pointing from medium 1 into medium 2, and  $\vec{K}$  denotes the surface current density on the interface.

The surface current density  $\vec{K}$  is defined as

$$\vec{K} = \lim_{t \rightarrow 0} (t\vec{J}) = \lim_{t \rightarrow 0} (t\sigma\vec{E}) \quad (5.47)$$

and vanishes unless the conductivity of medium at the interface is infinite. Here  $t$  is the thickness of an interface layer in which there is a finite volume current density  $\vec{J}$ . Since the Earth's conductivity is finite,  $\vec{K} = 0$ . Hence (5.47) reduces to

$$(\vec{H}_2 - \vec{H}_1) \times \vec{n} = 0 \quad (5.48)$$

So tangential components of  $\vec{E}$  and  $\vec{H}$  at interfaces of a plane homogeneously layered model of the Earth are continuous. Therefore, functions  $\vec{E}$  and  $\vec{H}$  in the plane homogeneously layered model of the Earth satisfy the necessary and sufficient condition for analytic continuation.

The Fourier transform of  $\vec{H}(x, y, z, \omega)$  with respect to  $x$  and  $y$ , the spectrum, is defined as

$$\vec{H}(k_x, k_y, z, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \vec{H}(x, y, z, \omega) \exp[i(k_x x + k_y y)] dx dy \quad (5.49)$$

and the inverse Fourier transform

$$\vec{H}(x, y, z, \omega) =$$

$$\frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{h}(k_x, k_y, z, \omega) \exp[-i(k_x x + k_y y)] dk_x dk_y \quad (5.50)$$

The three-dimensional Helmholtz equation (5.43) for  $\tilde{h}$  are reduced to one-dimensional equation for the spectrum,

$$\tilde{h}''(k_x, k_y, z, \omega) - \eta_n^2 \tilde{h}(k_x, k_y, z, \omega) = 0 \quad (5.51)$$

where the prime denotes differentiation over  $z$  and

$$\eta_n = \sqrt{(k_x^2 + k_y^2 + k_h^2)}, \quad \text{Re } \eta_n > 0 \quad (5.52)$$

A solution of equation (5.51) is

$$\tilde{h} = \tilde{h}_i \exp[+\eta_n z] + \tilde{h}_e \exp[-\eta_n z] \quad (5.53)$$

where  $\tilde{h}_i$  and  $\tilde{h}_e$  are respectively the Fourier transforms of fields at the Earth's surface of internal and external currents.

Assuming that the separation is successful, so that only the internal part is being continued,

$$h_z = h_i \exp[+\eta_n z] \quad (5.54)$$

$$\text{or } h_z(k_x, k_y, z, \omega) = h_i^z(k_x, k_y, 0, \omega) \exp[+\eta_n z] \quad (5.55)$$

(5.55) is an extension of (5.11) to the homogeneous half-space with finite conductivity. The inverse Fourier transform of (5.55) gives

$$H_z(x, y, z, \omega) = h_z^{-1}(k_x, k_y, z, \omega) \quad (5.56)$$

which is an extension of (5.12).

These equations have been applied to a half-space of conductivity  $10^{-3}$  S/m to continue the internal field at the Earth's surface, shown in Figures 5.3 and 5.4, downward to depths of 10, 20, 30 and 35 km. The results are shown in



Figures 5.17 to 5.24, which are almost indistinguishable from the maps at corresponding depths in Figures 5.5, 5.6, 5.9, 5.10, 5.13-5.16. Clearly the medium of resistivity 1000 ohm-m makes essentially no difference to the downward continuation.

#### 5.4 Downward Continuation of the Total Magnetovariation Field into the Earth (Zhdanov-Berdichevsky Method)

Berdichevsky and Zhdanov (1984) have proposed a method of analytic continuation of the electromagnetic field in terms of Stratton-Chu integral formulas.

Assume the electrical conductivity  $\sigma$  and the magnetic permeability  $\mu = \mu_0$  inside a domain  $V$  to be constants ( $\mu_0$  is the magnetic permeability in vacuum) and the sources of the electromagnetic field to be outside  $V$ . The Stratton-Chu integral formulas will be as follows (Berdichevsky and Zhdanov, 1984; Stratton, 1941),

$$\begin{aligned} & \iint_S \{ (\vec{n} \cdot \vec{E}) \nabla G + [\vec{n} \times \vec{E}] \times \nabla G + i\omega\mu[\vec{n} \times \vec{H}]G \} ds \\ &= \begin{cases} \vec{E}(\vec{r}') & M' \in V \\ 0 & M' \notin V \end{cases} \end{aligned} \quad (5.57)$$

$$\begin{aligned} \text{and } & \iint_S \{ (\vec{n} \cdot \vec{H}) \nabla G + [\vec{n} \times \vec{H}] \times \nabla G + \sigma[\vec{n} \times \vec{E}]G \} ds \\ &= \begin{cases} \vec{H}(\vec{r}') & M' \in V \\ 0 & M' \notin V \end{cases} \end{aligned} \quad (5.58)$$

where  $S$  is the surface enclosing  $V$ ,  $\vec{n}$  the unit vector of the inward normal and

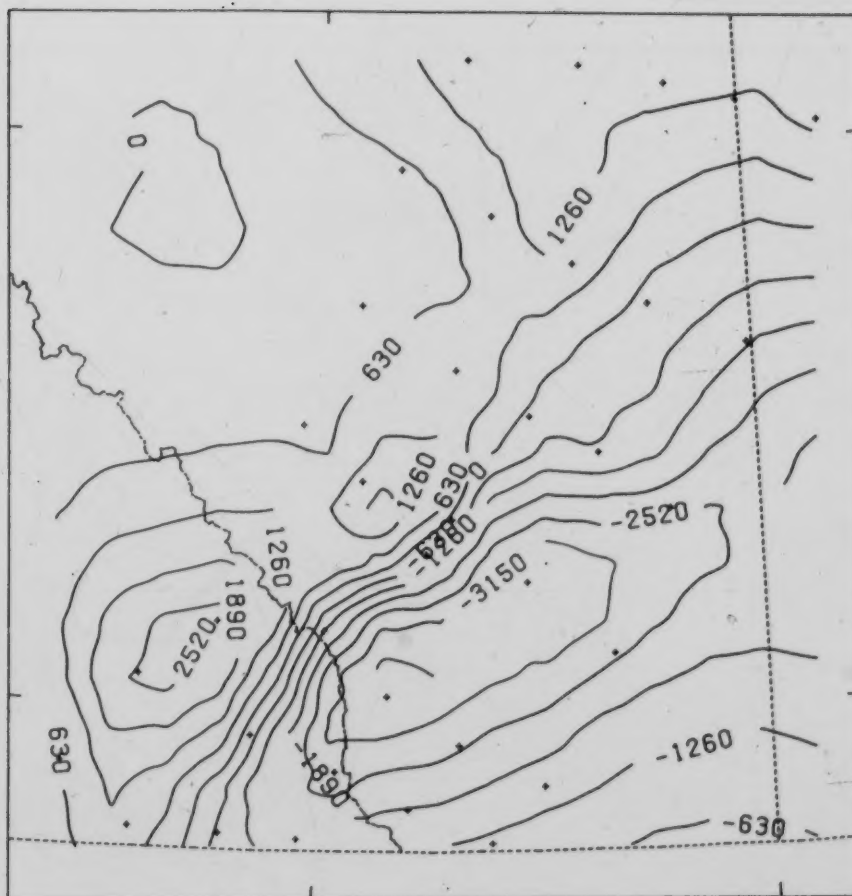


Figure 5.17 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: real Z component and depth 10 km

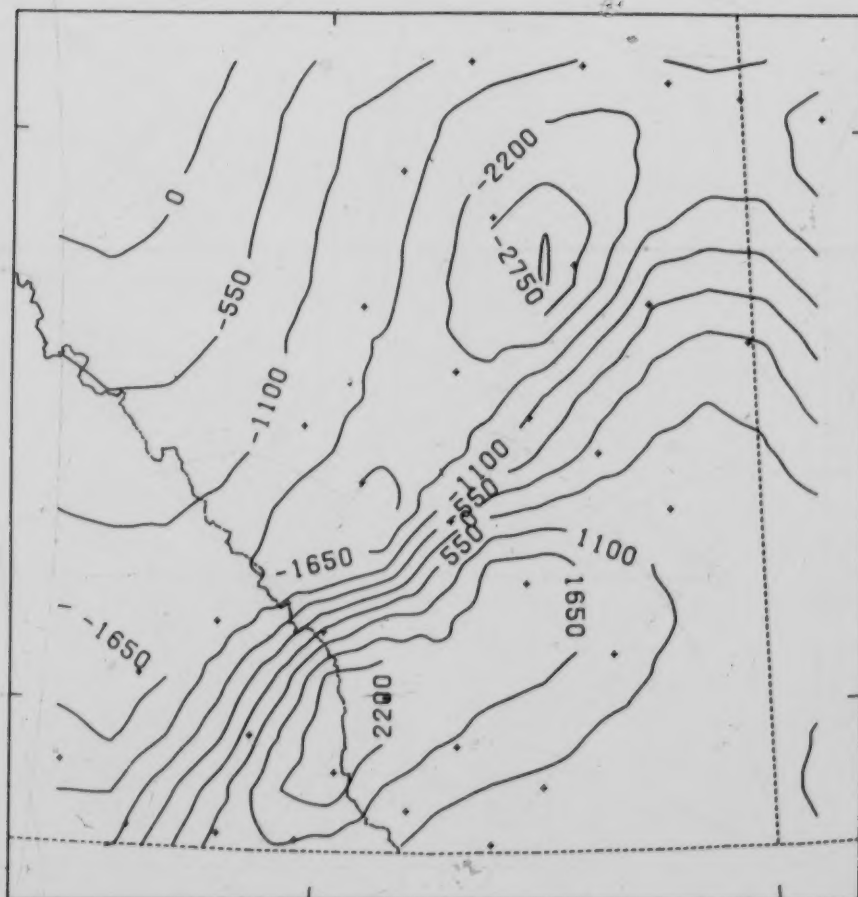


Figure 5.18 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: imaginary Z component and depth 10 km

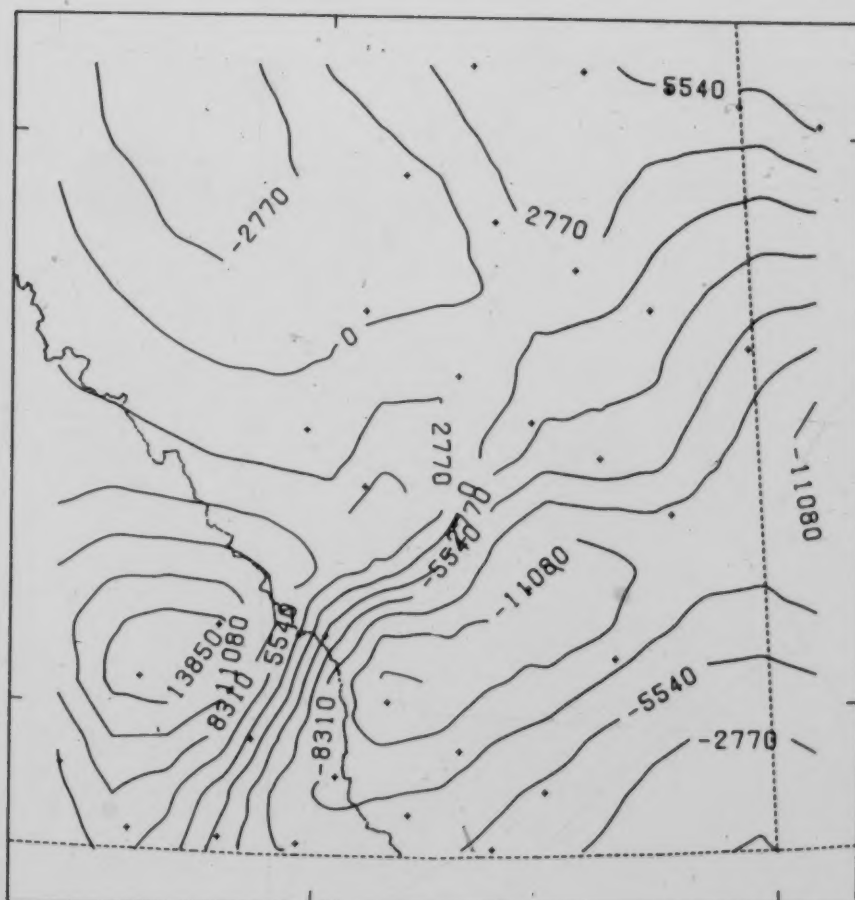


Figure 5.19 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: real Z component and depth 20 km

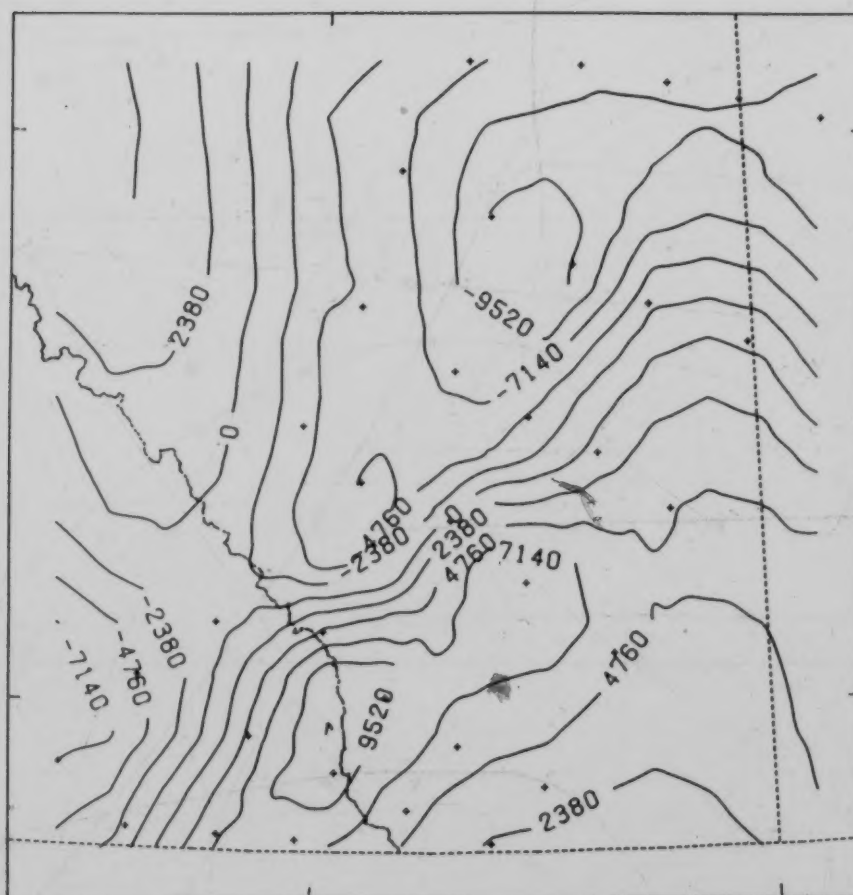


Figure 5.20 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: imaginary Z component and depth 20 km



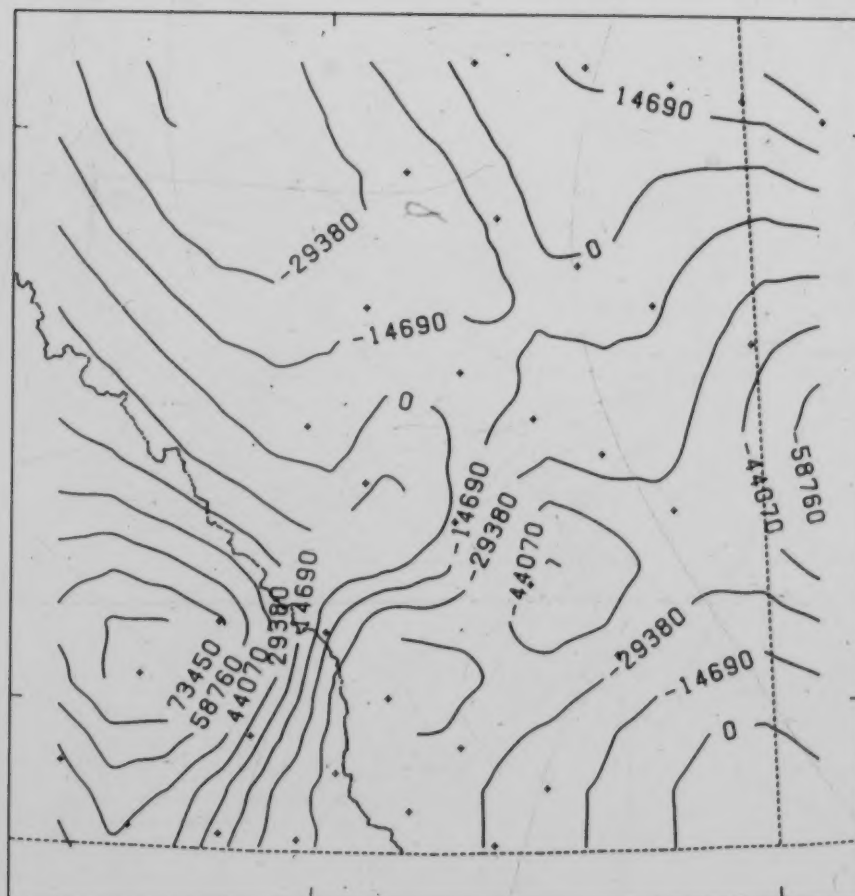


Figure 5.21 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: real Z component and depth 30 km

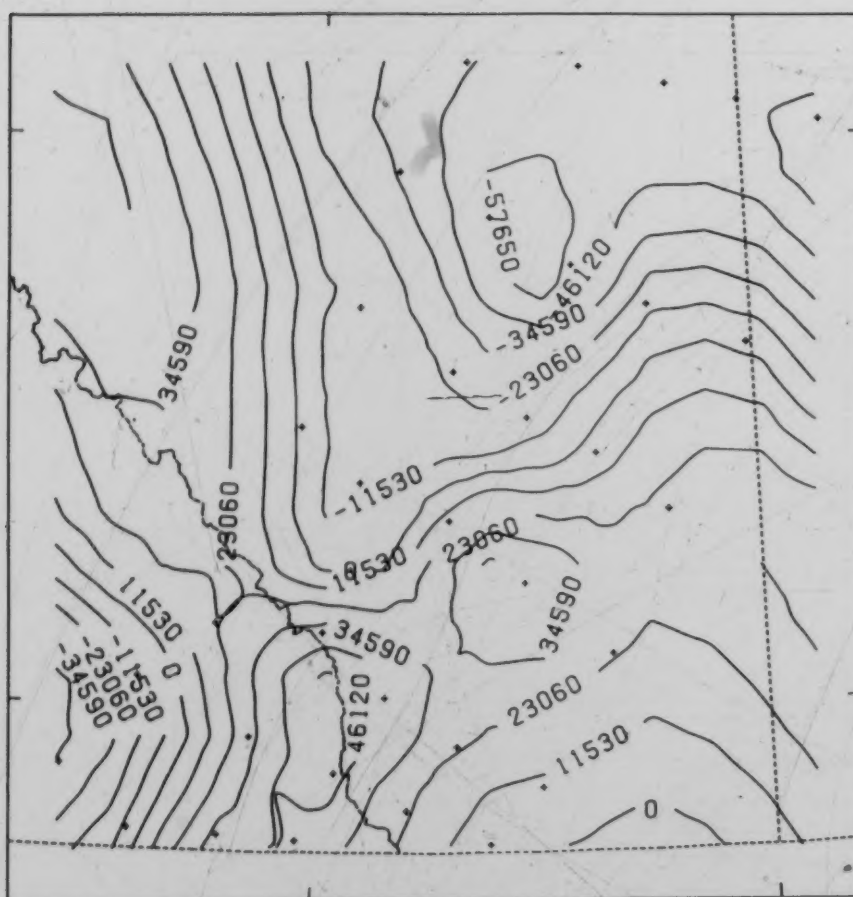


Figure 5.22 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: imaginary Z component and depth 30 km

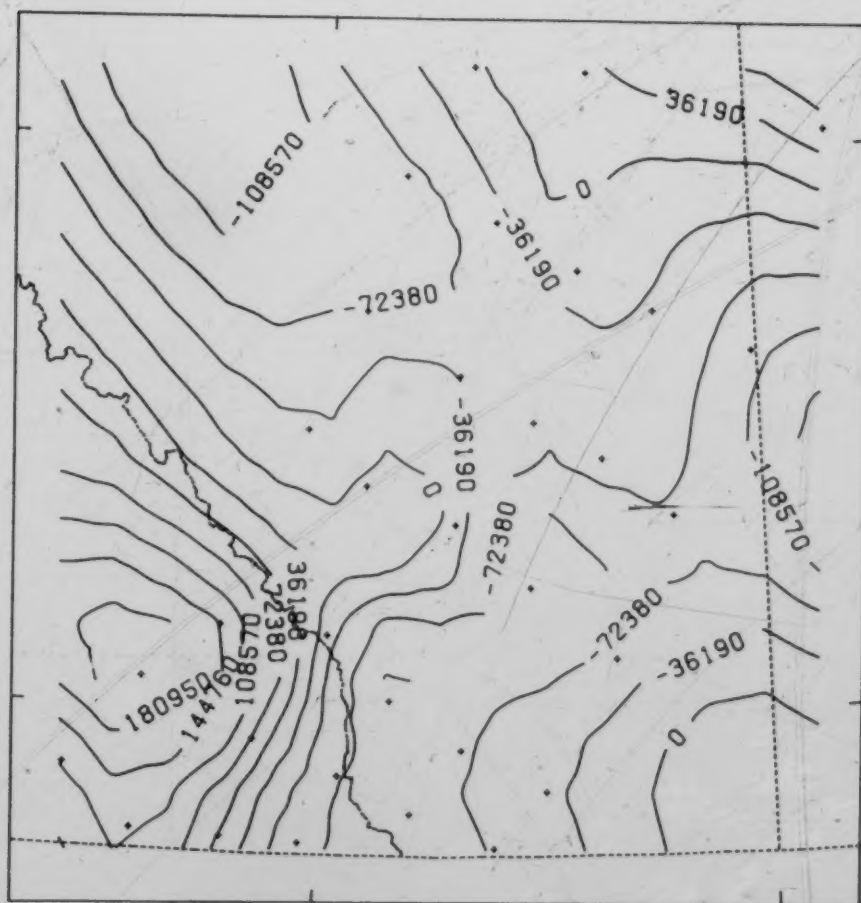


Figure 5.23 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: real Z component and depth 35 km

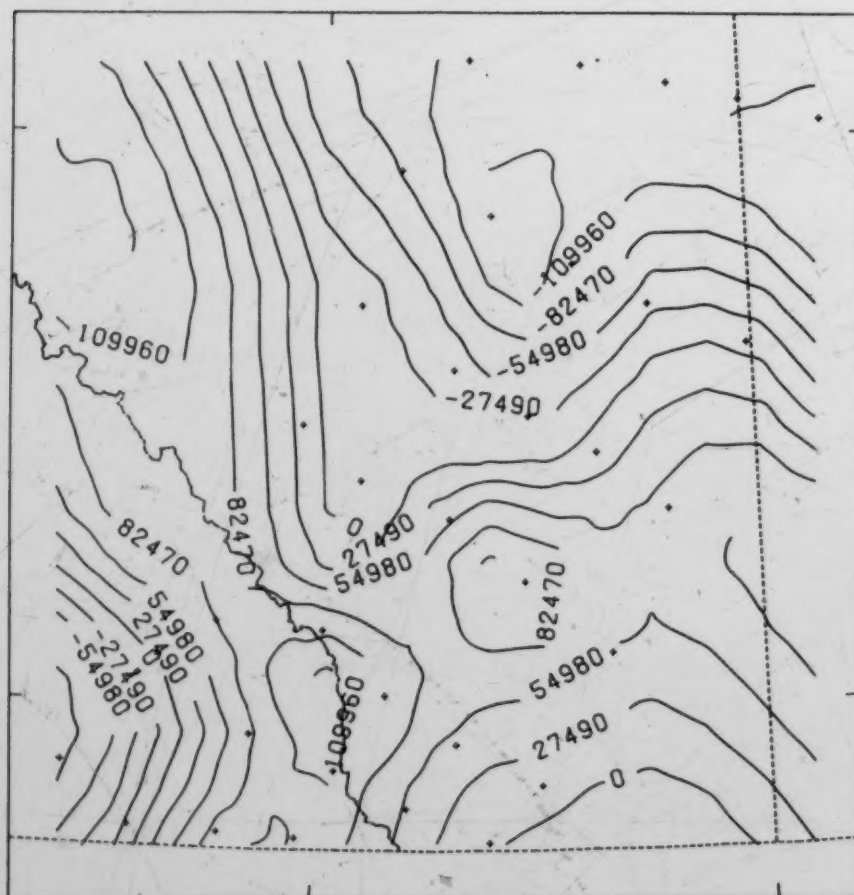


Figure 5.24 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m: imaginary Z component and depth 35 km

$$G = G(\vec{r}'/\vec{r}) = \frac{e^{ik|\vec{r}'-\vec{r}|}}{4\pi|\vec{r}'-\vec{r}|} \quad (5.59)$$

which is the fundamental Green function.

The Stratton-Chu formulas solve the problem of expressing the electromagnetic field  $\vec{E}$  and  $\vec{H}$  in the domain  $V$  in terms of its values over an enclosing surface.

Consider a horizontal layer  $V$  bounded by the Earth's surface  $S_0$  and a plane  $S_1$ . Assume the constant conductivity  $\sigma_n$  and magnetic permeability  $\mu$  to be known,  $\vec{E}^0$  and  $\vec{H}^0$  to be specified on the Earth's surface, and the field source to be outside  $V$ . For the field at  $M' \in V$ , we have from (5.57) and (5.58),

$$\begin{aligned} \vec{E}(\vec{r}') = & \iint_{S_0} \{ (\vec{n} \cdot \vec{E}) \nabla G + [\vec{n} \times \vec{E}] \times \nabla G + i\omega\mu[\vec{n} \times \vec{H}]G \} ds \\ & + \iint_{S_1} \{ (\vec{n} \cdot \vec{E}) \nabla G + [\vec{n} \times \vec{E}] \times \nabla G + i\omega\mu[\vec{n} \times \vec{H}]G \} ds \end{aligned} \quad (5.60)$$

$$\begin{aligned} \vec{H}(\vec{r}') = & \iint_{S_0} \{ (\vec{n} \cdot \vec{H}) \nabla G + [\vec{n} \times \vec{H}] \times \nabla G + \sigma[\vec{n} \times \vec{E}]G \} ds \\ & + \iint_{S_1} \{ (\vec{n} \cdot \vec{H}) \nabla G + [\vec{n} \times \vec{H}] \times \nabla G + \sigma[\vec{n} \times \vec{E}]G \} ds \end{aligned} \quad (5.61)$$

where

$$G = G(\vec{r}'/\vec{r}) = \frac{e^{ik_n|\vec{r}'-\vec{r}|}}{4\pi|\vec{r}'-\vec{r}|} \quad (5.62)$$

with  $k_n = \sqrt{(i\omega\mu\sigma_n)}$  and  $\text{Re } k_n > 0$ . After a series of transformations, Berdichevsky and Zhdanov (1984) obtained the following formulas,



$$E_x(\vec{r}') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ e_x^0 \cosh(\eta_n z') + i\omega\mu h_y^0 \frac{\sinh(\eta_n z')}{\eta_n} \right] \times \\ \exp[-i(k_x x' + k_y y')] dk_x dk_y \quad (5.63)$$

$$E_y(\vec{r}') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ e_y^0 \cosh(\eta_n z') - i\omega\mu h_x^0 \frac{\sinh(\eta_n z')}{\eta_n} \right] \times \\ \exp[-i(k_x x' + k_y y')] dk_x dk_y \quad (5.64)$$

$$E_z(\vec{r}') = \frac{i}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [k_x e_x^0 + k_y e_y^0] \frac{\sinh(\eta_n z')}{\eta_n} \times \\ \exp[-i(k_x x' + k_y y')] dk_x dk_y \quad (5.65)$$

$$H_x(\vec{r}') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ h_x^0 \cosh(\eta_n z') + (\sigma_n e_y^0 - i k_x h_z^0) \frac{\sinh(\eta_n z')}{\eta_n} \right] \times \\ \exp[-i(k_x x' + k_y y')] dk_x dk_y \quad (5.66)$$

$$H_y(\vec{r}') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ h_y^0 \cosh(\eta_n z') - (\sigma_n e_x^0 + i k_y h_z^0) \frac{\sinh(\eta_n z')}{\eta_n} \right] \times \\ \exp[-i(k_x x' + k_y y')] dk_x dk_y \quad (5.67)$$

$$H_z(\vec{r}') = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ h_z^0 \cosh(\eta_n z') + i(k_x h_x^0 + k_y h_y^0) \frac{\sinh(\eta_n z')}{\eta_n} \right] \times \\ \exp[-i(k_x x' + k_y y')] dk_x dk_y \quad (5.68)$$

where  $\vec{e}^0$ ,  $\vec{h}^0$  are the spectra of the fields  $\vec{E}^0$ ,  $\vec{H}^0$  in the wavenumber-frequency domain.

Note that (5.68) involves only three components of the magnetic field without any components of electric field. Hence it is applicable to the magnetovariation array study. Let us take a close look at (5.68). Putting

$$F_z(k_x, k_y, z, \omega) = h_z^0 \cosh(\eta_n z) + i(k_x h_x^0 + k_y h_y^0) \frac{\sinh(\eta_n z)}{\eta_n} \quad (5.69)$$

one finds

$$F_z(x, y, z, \omega) = F_z^1(k_x, k_y, z, \omega) \quad (5.70)$$

From (5.50) and  $\nabla \cdot \vec{H}^0 = 0$ ,

$$ik_x h_z^0 + ik_y h_z^0 = h_z^0 \quad (5.71)$$

From (5.53),

$$h_z = h_{1z}^0 \exp[+\eta_n z] + h_{2z}^0 \exp[-\eta_n z] \quad (5.72)$$

$$\text{and } h_z^0 = (h_{1z}^0 + h_{2z}^0) \quad (5.73)$$

$$h_z^0 = \eta_n (h_{1z}^0 - h_{2z}^0) \quad (5.74)$$

Combining (5.71), (5.73) and (5.74) with  $\sinh(\eta_n z) = (\exp[+\eta_n z] - \exp[-\eta_n z])/2$  and  $\cosh(\eta_n z) = (\exp[+\eta_n z] + \exp[-\eta_n z])/2$ , equation (5.69) gives

$$F_z = h_{1z}^0 \exp[+\eta_n z] + h_{2z}^0 \exp[-\eta_n z] \quad (5.75)$$

which is the same as equation (5.72), that is,  $F_z = h_z$ . Note that the arguments in  $F_z$  and  $h_z$  are omitted. Hence equation (5.68) is valid for the Z-component of the total magnetovariation field. However, if the field only involves the internal field, then  $h_{2z}^0 = 0$  and (5.75) reduces to (5.56). The results from (5.68) are shown in Figures 5.25 to 5.32.

### 5.5 Depth Estimation of the SABC Conductor

From those maps presented above, Figures 5.3-5.32 (except 5.25 -5.32), we can see that the patterns of anomalies for the SABC conductor continue downward with

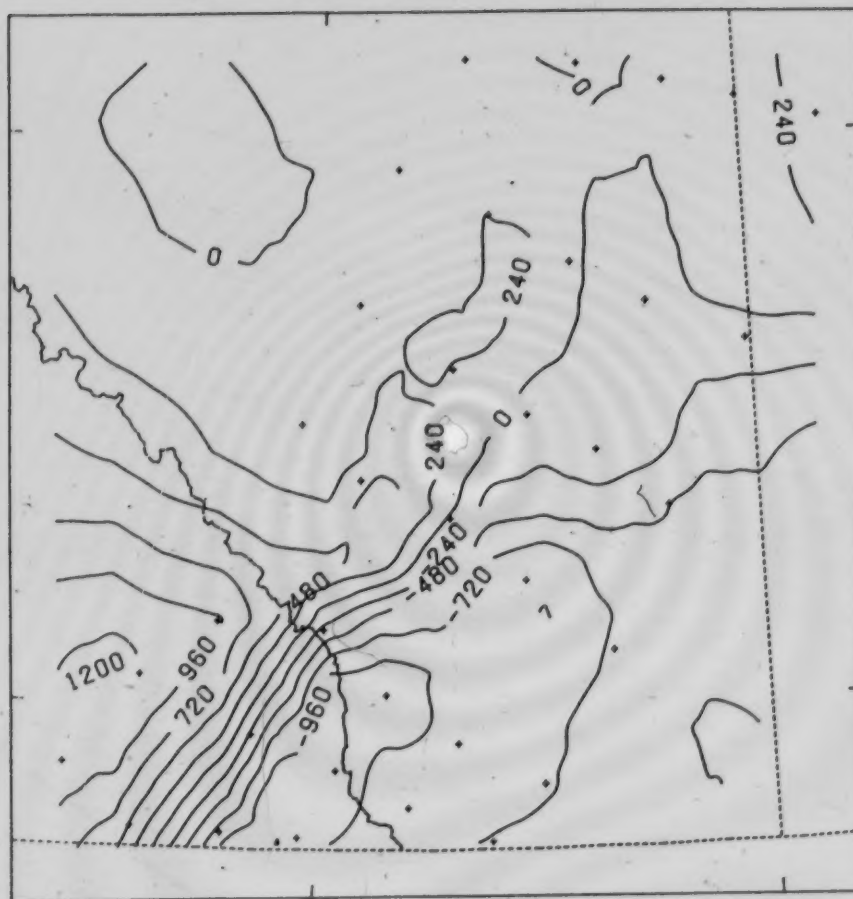


Figure 5.25 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): real Z component and depth 10 km

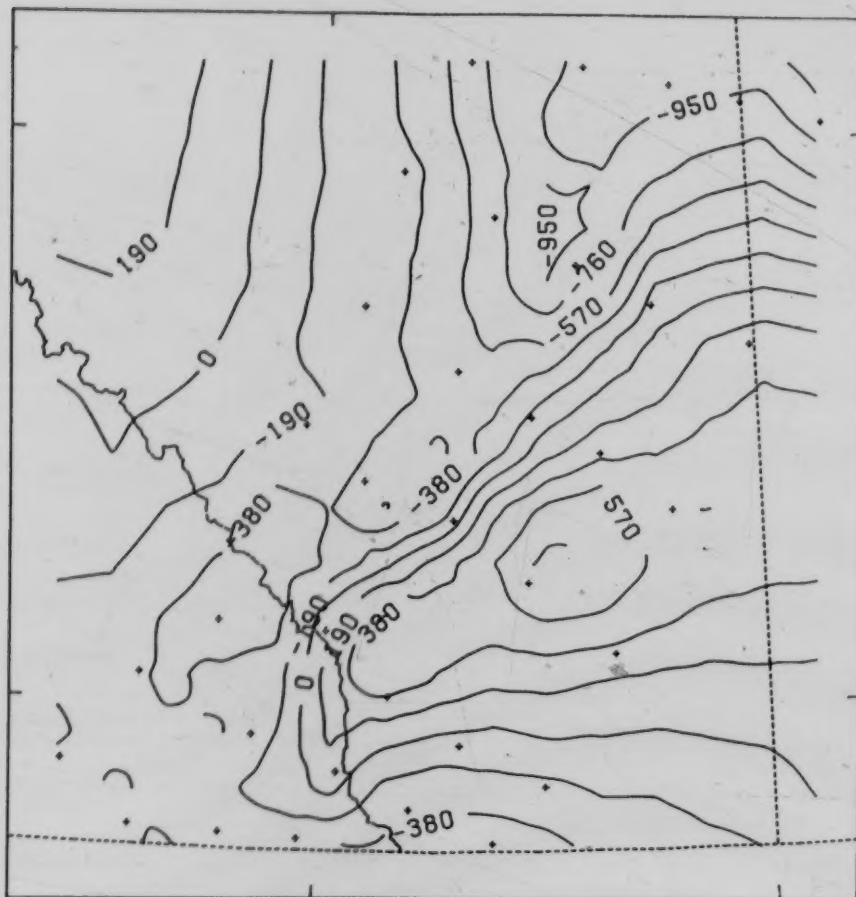


Figure 5.26 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): imaginary Z component and depth 10 km

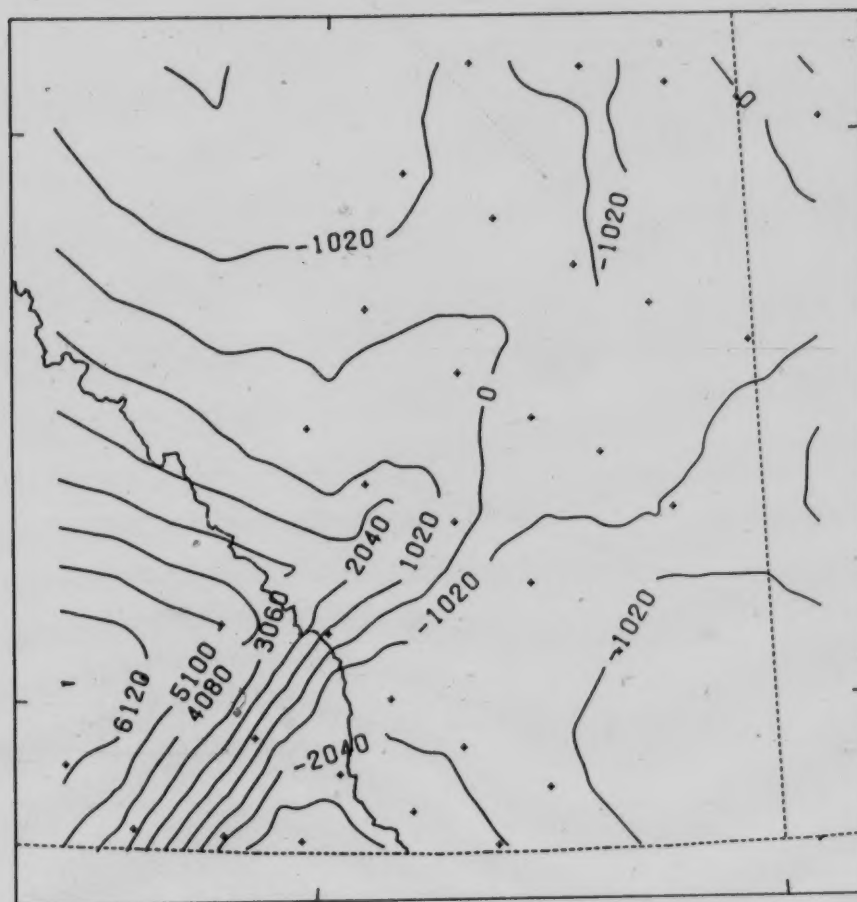


Figure 5.27 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): real Z component and depth 20 km



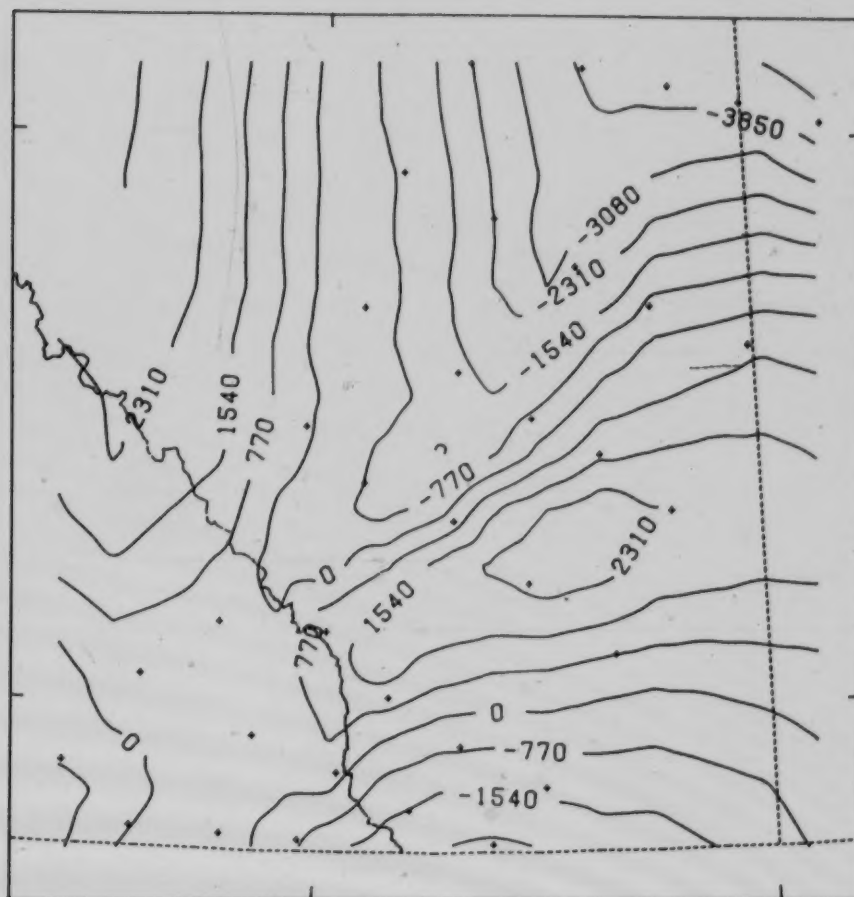


Figure 5.28 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): imaginary Z component and depth 20 km

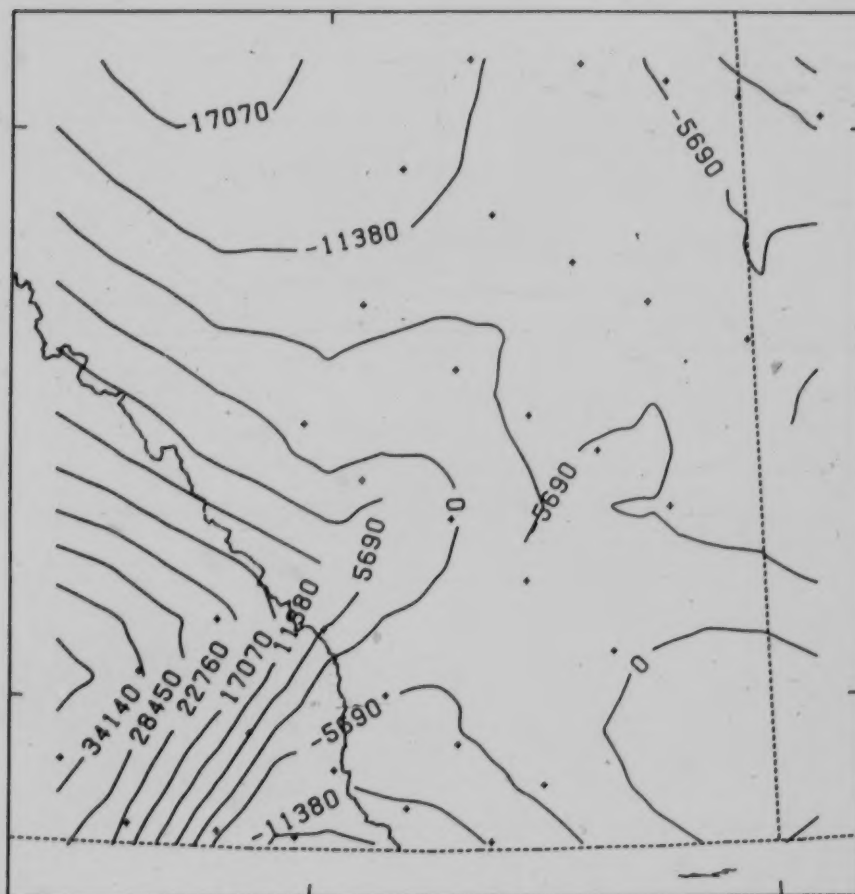


Figure 5.29 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): real Z component and depth 30 km

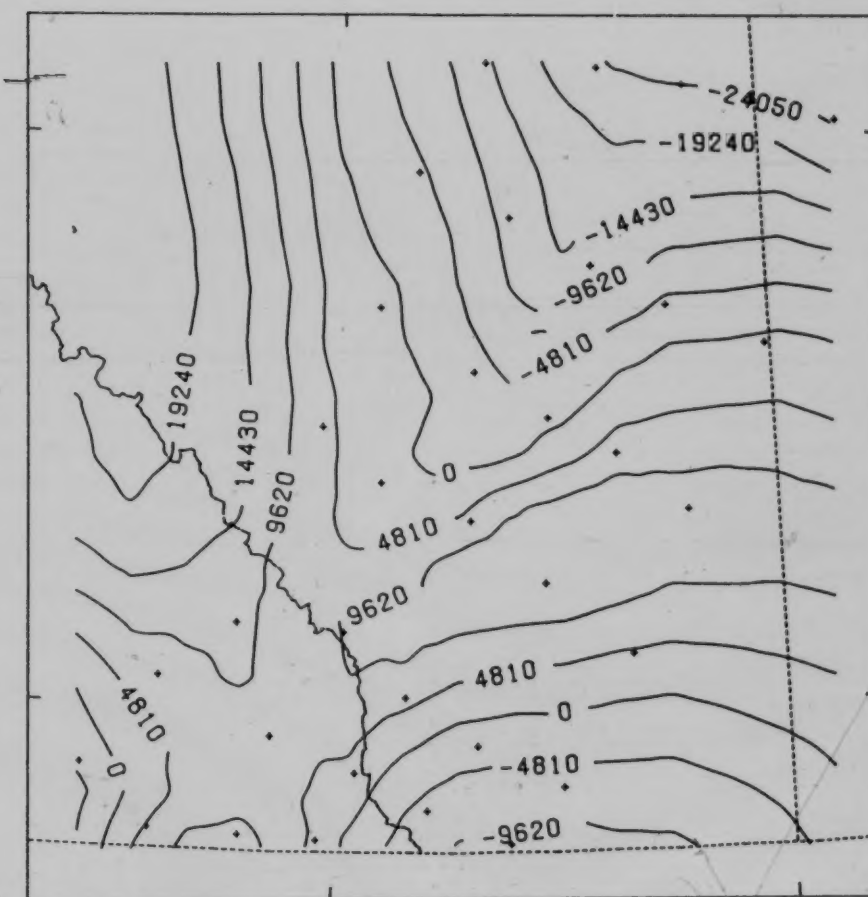


Figure 5.30 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): imaginary Z component and depth 30 km

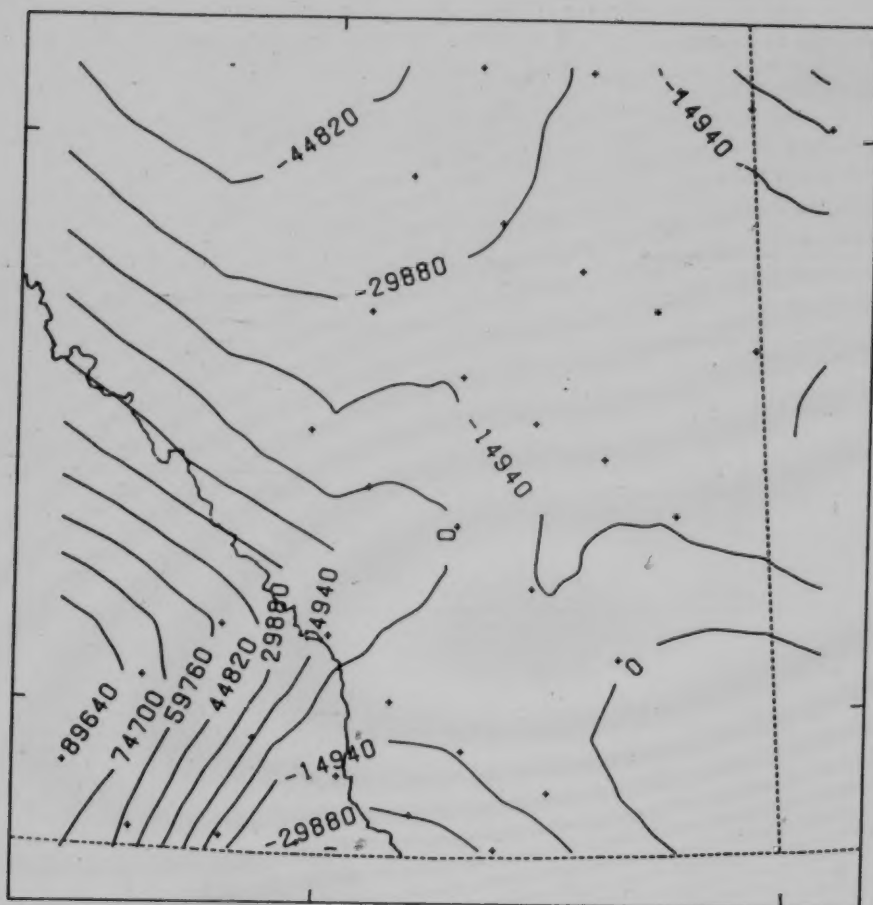


Figure 5.31 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): real Z component and depth 35 km

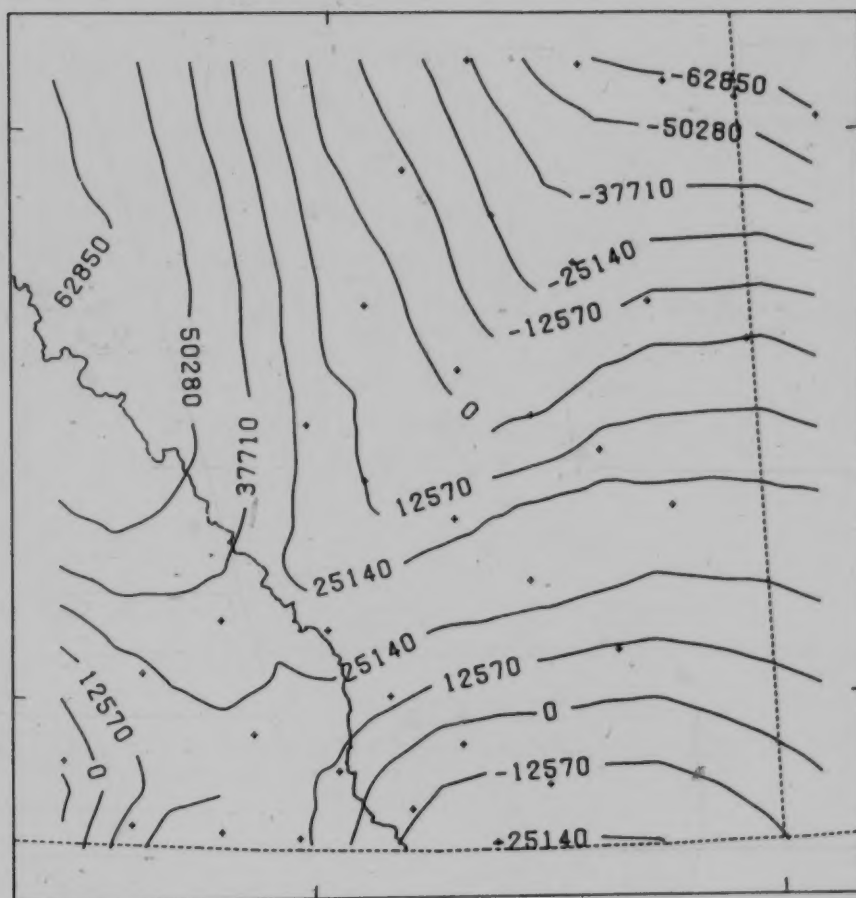


Figure 5.32 Downward continuation of the internal field into the homogeneous half-space with conductivity  $10^{-3}$  S/m (Zhdanov-Berdichevsky method): imaginary Z component and depth 35 km



little change from the surface to a depth of 20 km. A change in the pattern starts at a depth of 25 km. The lines of zero value in both real and imaginary Z maps have changed their directions. At depth 35 km the anomaly of the SABC conductor in the central part of the array has disappeared. Therefore, the depth of the SABC conductor is about 25 to 30 km in the central section and looks deeper at both ends. Further discussions about the location of the SABC conductor in the Earth will be given in 6.1.

## 6. RELATION OF THE SABC CONDUCTOR TO GEOPHYSICS AND GEOLOGY OF THE REGION

### 6.1 Location of the SABC Conductor in the Earth

A partially reversed seismic refraction profile about 450 km long, from Sparwood to the Highland valley in southern British Columbia near latitude  $50^{\circ}\text{N}$ , was reported by Cumming, Clowes and Ellis (1979). The refraction data indicate that the crustal thickness increases eastward from about 30 km below the Highland Valley to about 40 km beneath the Purcell Geanticline and the Rockies. The M-discontinuity shows undulations of about 165 km wavelength and a few kilometers amplitude. The upper mantle  $P_n$  velocity is estimated to be 7.8 km/s. The SABC conductor crosses the profile between CRA and ELK (near Sparwood), where the M-discontinuity is at a depth of 40 km. Hales and Nation (1973) interpreted two long seismic refraction profiles in the south-easterly direction from a shot point in Greenbush Lake, near Revelstoke, British Columbia and indicated a crustal thickness of about 37 km in the Rockies near the British Columbia border with Idaho and Montana, between stations CRS and KAS. This profile crosses the Sparwood-Highland Valley profile of Cumming et al. near station CRA. Chandra and Cumming (1972) interpreted a seismic refraction profile about 700 km long, from Greenbush Lake to Swift Current, Saskatchewan, along  $50.5^{\circ}\text{N}$ . The results indicate that upper mantle velocities vary along the

profile from about 8.0 km/s west of the Rockies to 8.3 km/s in the region between Vulcan and Suffield in southern Alberta, and the thickness of crust increases eastward from 35 km under the Intermontane Belt to about 45 km under the Alberta plains, and a crustal root under the Rocky Mountains of about 5 km exists. The SABC conductor crosses the profile between WIR and CAH.

Richards and Walker (1959) measured the thickness of the Earth's crust east of Calgary, southern Alberta by the seismic refraction method. The profile ran approximately in a north-south direction across the SABC conductor. They observed a Conrad discontinuity and Mohorovicic discontinuity at depths of 29 and 43 km, with velocities of 7.2 and 8.2 km/s, respectively. Kanasewich and Cumming (1965) detected the Conrad discontinuity near Lomond in southern Alberta at a depth of 34 km, assuming a mean velocity of 6.37 km/s, by the seismic reflection method. Later, Professor D. H. Hall proposed to refer to the upper discontinuity as the Riel (R) discontinuity (Clowes et al., 1968). Two deep seismic reflection profiles in the north-south direction between Vauxhall and Carbon, east of Calgary, southern Alberta, have been studied in detail (Clowes et al., 1968; Clowes and Kanasewich, 1972). The results show that the M discontinuity varies in depth from 47 to 38 km while the R discontinuity changes from 37 to 26 km from the top of the Paleozoic. Both show a relief of as much as 9 km over a horizontal distance of only 40 km. The R

discontinuity is a sharp transition in a depth range less than 0.5 km and with a velocity increase from 6.5 to 7.2 km/s. The location of the northern profile is near Rosebud. At the northern end of the northern profile the depth to the Riel discontinuity is about 31 km from the top of the Paleozoic. The north end of the northern profile touches the southeastern edge of the central part of the SABC conductor.

Vozoff and Ellis (1966) presented results of magnetotelluric soundings at Vulcan in southern Alberta, which is located over the Precambrian Rift proposed by Kanasewich and his colleagues (Kanasewich, 1968; Kanasewich et al., 1969). A decrease of resistivity from 1000 to 30 ohm-meters appears at about 35 km while the depth to the Riel discontinuity in the southern profile is 34 km from the top of the Paleozoic. Caner et al, (1969) found a conducting zone (30-50 ohm-meters at depth 30-35 km) at Fernie, British Columbia and at Pincher, Alberta by MT sounding studies. Fernie is right on the edge of the SABC conductor.

In Chapter 5 it has been shown, by means of downward continuation of the magnetovariation fields, that the currents in the SABC conductor flow in the depth range 25-30 km. The upper surface of the highly conductive body lies therefore in the lower crust, just above the Riel discontinuity and the body probably continues to the upper mantle, but its thickness is undetermined. We say that the SABC conductor lies in the lithosphere.

## 6.2 Relation between the SABC Conductor and the Precambrian Rift in Southern Alberta

A Precambrian rift below flat-lying Paleozoic sediment (2.5 km) to an average depth of 30 km was detected by studies of seismic reflection, gravity and magnetic data (Kanasewich, 1968; Kanasewich et al., 1969). The rift showed dips up to 20°, and faults with vertical displacements of as much as 5 km (Clowes and Kanasewich, 1972). Model calculations, with the use of a relation between seismic velocities and rock densities, supported the hypothesis that the rift was filled with low-density, nonmagnetic, Precambrian sediments. The feature was traced by Kanasewich (1968), by means of gravity and magnetic anomalies, for several hundred kilometers across Alberta, the Rocky Mountains and British Columbia to the Idaho Panhandle. The position of the rift as inferred by Kanasewich and his colleagues is shown in Figure 6.1 (broken lines).

The SABC conductor was detected and mapped in earlier work with magnetovariation fields (Cochrane and Hyndman, 1970; Dragert and Clarke, 1977; Gough et al., 1982). However, the relation between the conductor and the Precambrian rift was not clear.

The present thesis has studied the SABC conductor in detail by means of data from the 1983 and 1981B arrays through various techniques of analysis of its magnetovariation fields: magnetograms, FT anomaly maps, transfer functions (induction arrow and hypothetical event



presentations), and downward continuation methods. Combining the results obtained from the above techniques, we get the location of the SABC conductor on the Earth's surface, shown in Figure 6.1 (solid lines). This is a very large elongated conductor. Figure 6.1 shows that the SABC conductor and the Precambrian rift coincide in position only near  $49^{\circ}\text{N}$ . They differ also in depth: the Precambrian rift lies beneath the flat-lying Paleozoic sediment (2.5 km from the surface) and continues to the M-discontinuity, and so it lies in the Earth's crust. However, the SABC conductor lies in the lithosphere. It appears that the conductor and rift are not associated.

### 6.3 Relation of the SABC Conductor to the North American Central Plains (NACP) Conductor

An array of 42 GR magnetometers in 1967 detected magnetovariation anomalies at stations south of the Black Hills of South Dakota (Reitzel et al., 1970; Porath et al., 1970). A second array in 1969, with 46 GR magnetometers, established the existence of a conductor confined to a long, narrow region extending from the Black Hills northward to the United States-Canada border (Camfield et al., 1971). A third array, with 41 GR magnetometers, was operated in 1972 to map the conductor southward from the Black Hills and northward into Canada (Alabi et al., 1975). Alabi et al. described a detailed study of the NACP conductor. The conductor runs from beneath the exposed Precambrian of the

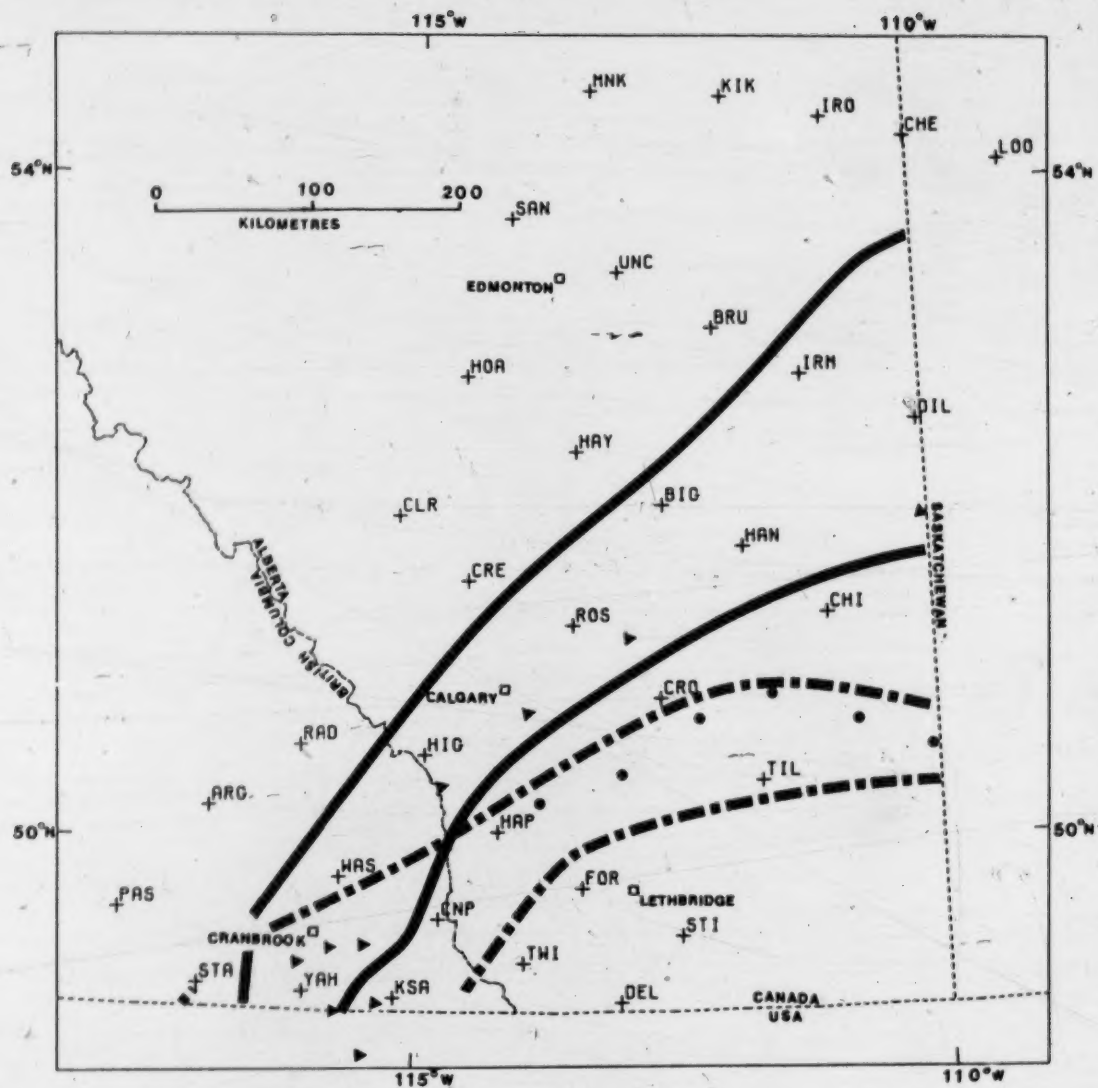


Figure 6.1 Location of the Southern Alberta-British Columbia (SABC) conductor and the Precambrian rift proposed by Kanasewich and his colleagues (1968) on the Earth's surface.

southern Rockies in Wyoming, to the northeast, skirting the eastern side of the Black Hills, then striking northward roughly along longitude  $104^{\circ}\text{W}$  in the United States. In Canada the conductor passes near Regina, bends northwestward to the vicinity of Saskatoon, then turns northward again and reaches nearly to the edge of the exposed basement rocks of the Churchill Province of the shield. Seven magnetometers recorded magnetovariation fields on a northwest-southeast line 280 km long in north-central Saskatchewan during July 1981 in order to test the hypothesis advanced in 1975 by Alabi et al that the NACP conductor links with the Wollaston Domain in the exposed Precambrian shield of Saskatchewan. The results have shown that the NACP conductor passes through the Rottenstone-La Ronge Magmatic Belt, the immediate east of the Wollaston Domain (Handa and Camfield, 1984). Gupta et al (1985) investigated conductive structures in north-eastern Manitoba and part of the North-west Territories by using Canadian geomagnetic data collected during the International Magnetospheric Study (IMS) and found a Gillam-Back (GIM-BKC) conductor in Manitoba. They used the surface geological trends in Saskatchewan and Manitoba as a guide to link the NACP conductor with the Gillam-Back conductor and indicated that the conductor extends further eastward under Hudson Bay. Figure 6.2 shows the whole NACP conductor.

Recently, Green et al (1985) have extended the knowledge of the exposed Canadian Precambrian Shield beneath



Figure 6.2 Location of the North American Central Plains conductive body (broad shaded area). In the Churchill Province the Wathaman-Chipewyan Batholith lies to the north and the bulk of the Kisseynew gneiss belt to the south of the conductor. The triangles locate the array of 41 magnetometers from Alabi, Camfield & Gough (1975) and the permanent observatory at Meanook, the crosses locate an array of seven magnetometers established by Handa & Camfield (1984) and dots represents the IMS stations. Solid lines show the SABC conductor discussed in the present thesis. Modified from Gupta et al., 1985.



the Phanerozoic Williston basin of south-central Canada and the north-central United States. A new tectonic map, shown in Figure 6.3, was constructed. Figure 6.3 shows that the principal geological units possess prominent parallel trends. Boundaries of the units in the north strike in an E-W direction, whereas to the south they trend NE-SW and finally N-S. The NACP conductor follows the Reindeer-South Indian Lake belt.

It is interesting to speculate that the SABC conductor may link to the NACP conductor north of Saskatoon, where they have been mapped within about 100 km of one another (see Figure 6.2). Burwash and Culbert (1976) have discussed regional lineaments in the Precambrian basement of western Canada (Figure 6.4), based on geophysical and tectonic data. They combined these basement lineaments with the results of their analysis of multivariate geochemical and mineral patterns and extended the tectonic units of the exposed Churchill Shield southward to western Canada basin. It is clear from Figure 6.4 that the structural trends are NE-SW in northern Saskatchewan, but then bifurcate in central-southern Saskatchewan. The NACP conductor follows the southeastern branch, while the SABC might follow the southwestern one. The NACP has been proposed as a Proterozoic plate boundary by Camfield and Gough (1977). The SABC conductor could possibly trace another Proterozoic suture. Ages of samples from deep wells into the Precambrian basement in the western Canadian sedimentary basin show that



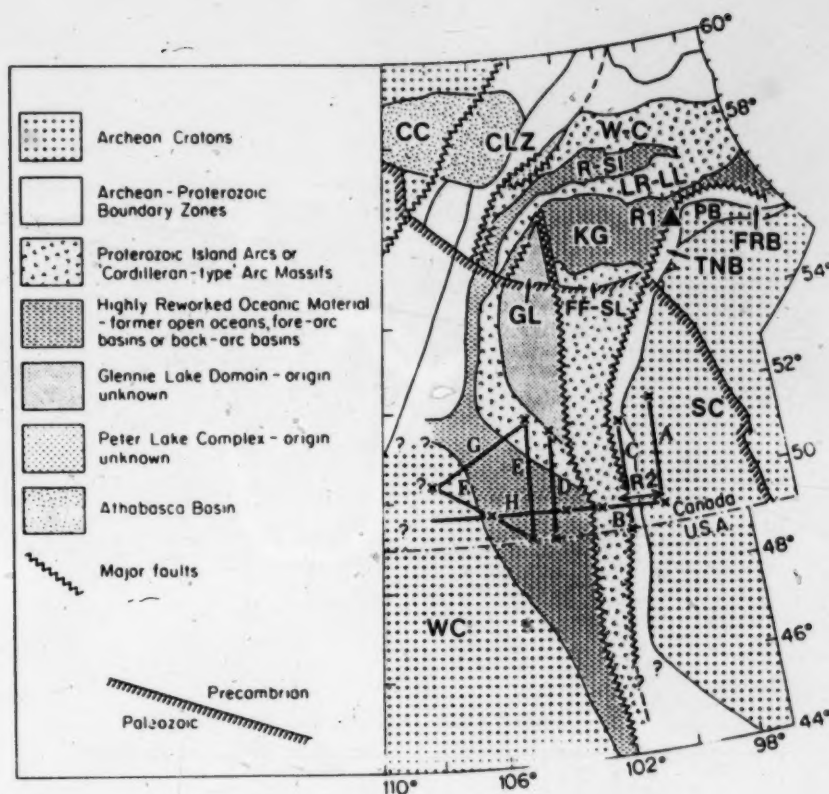


Figure 6.3 Tectonic provinces of the Canadian Shield in Manitoba, Saskatchewan, North Dakota and parts of South Dakota, Montana and Wyoming. CC—Churchill craton, CLZ—Cree Lake zone, FF-SL—Flin Flon-Snow Lake belt, FRB—Fox River belt, GL—Glennie Lake domain, LR-LL—La Ronge-Lynn Lake belt, KG—Kisseynew belt, PB—Pikwitonei granulite, R-SI—Reindeer-South Indian Lakes belt, SC—Superior craton, TNB—Thompson belt, W-C—Wathaman-Chipaweyan batholith, WC—Wyoming craton. Modified from Green et al., 1985.

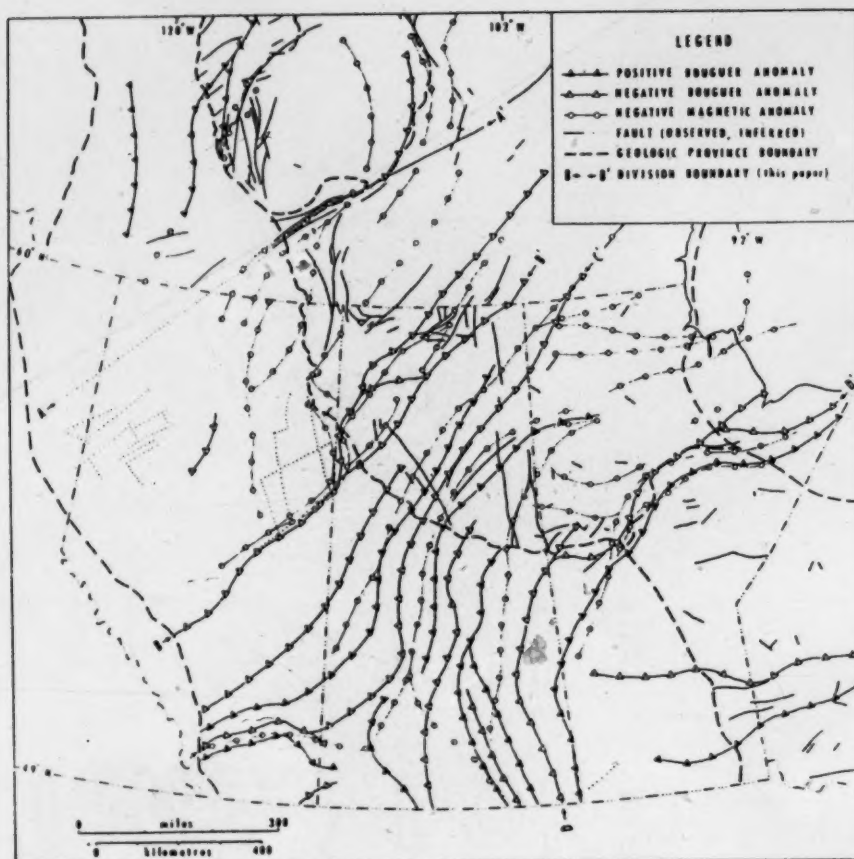


Figure 6.4 Lineaments in the western Canada. From Burwash and Culbert, 1976.

the western Churchill and Wyoming provinces are linked together as a single craton, having a late Archean or reworked Archean crust (Burwash et al., 1962; Frost and Burwash, 1986). The SABC conductor thus is not a boundary between Churchill and Wyoming cratons.

#### 6.4 Possible Causes of the High Conductivity in the SABC Conductor

As noted in the Introduction, there are four types of conduction mechanisms which may cause high conductivity in the Earth. High conductivity may be associated with mineralization containing metallic sulphides or graphite (Camfield et al., 1971; Camfield and Gough, 1977); with hydrated rocks (Hyndman and Hyndman, 1968; Cochrane and Hyndman, 1974; Nekut et al., 1977); with serpentinized rocks in buried oceanic crust (de Beer et al., 1982); with a fracture zone containing interconnected saline water (Gough, 1986b); or with partial melting.

Laboratory measurements on solid and molten basalt indicate that the conductivity increases by about a factor of  $10^2$  in the melting range (Khitarov and Slutskiy, 1965; Presnall et al., 1972). For granites the increase in conductivity is up to about 3 orders of magnitude (Lebedev and Khitarov, 1964). Partial melting in dry rocks requires high temperatures of the order of 1000°C or higher. A small amount of water lowers the required temperature somewhat. Recently, terrestrial heat flow in Alberta basin have been

studied on the basis of 55,244 bottom hole temperatures values from 28,260 petroleum exploration wells (Majorowicz et al, 1984; 1985). It was found that a difference in heat flow exists between Paleozoic and Mesozoic+Cenozoic strata, and increase with depth in water recharge areas and decrease with depth in discharge areas. The results show that upward and downward water movement in Mesozoic and Cenozoic strata strongly influences the heat flow pattern. Figure 9 of Majorowicz et al (1985) shows the heat flow map of Alberta in Paleozoic strata (see Figure 6.5). A high geothermal heat flow zone ( $>100 \text{ mW/m}^2$ ) strikes NW-SE direction along the Rocky Mountain foothills (no data available in Rocky Mountains), which is parallel to the NR conductor. Another large high geothermal heat flow zone strikes in the NE-SW direction, which coincides with the direction of the SABC conductor in Alberta. The high geothermal heat flow in the Paleozoic strata might come from partially melted rock. The difference of the heat flow pattern between Paleozoic and Mesozoic+Cenozoic strata is due to the redistribution of the heat by downward and upward groundwater movement in Mesozoic and Cenozoic strata. However we need further evidence to support the existence of partial melt in the SABC conductor.

Hyndman and Hyndman (1968) first noted that water saturated rock in the lower crust may provide high conductivity. There are different ways in which water can do this (Greenhouse and Bailey, 1981): firstly, free water supports electrolytic conduction through pore and fracture



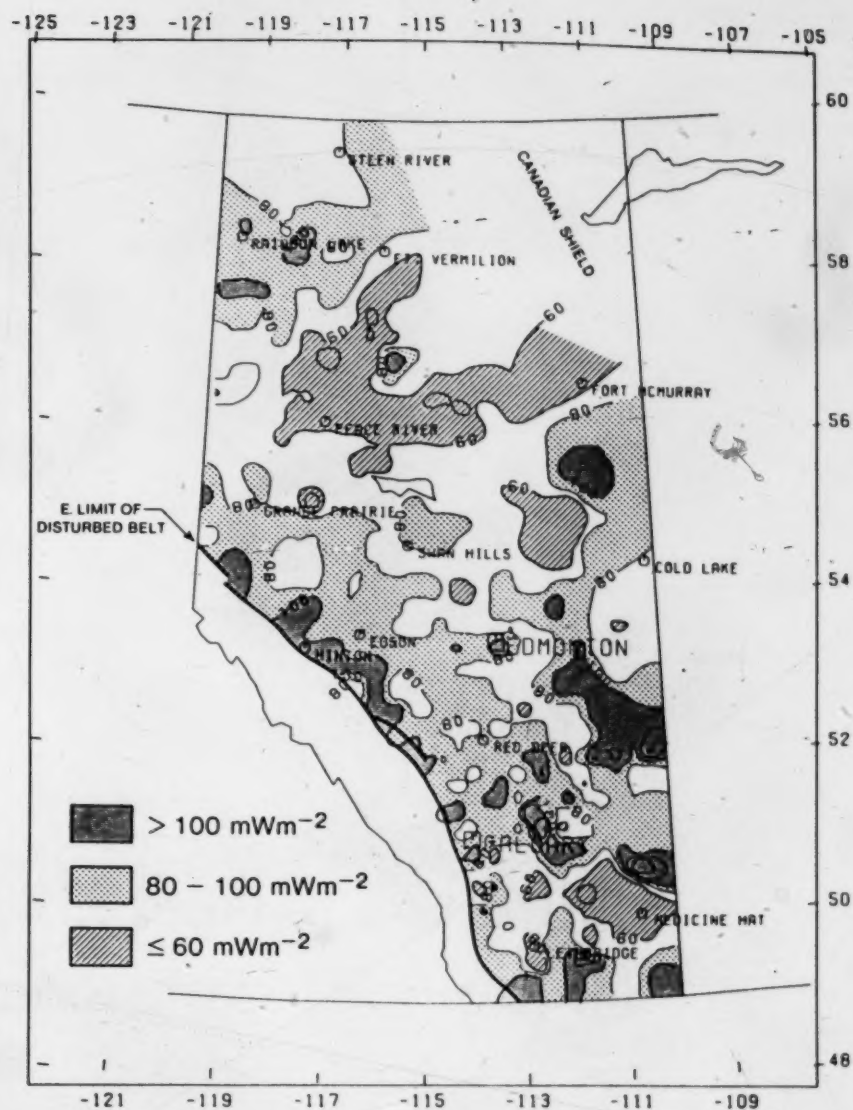


Figure 6.5 Heat flow map of Alberta based on estimated heat flow values (in  $\text{mW/m}^2$ ) below the Paleozoic surface (Paleozoic strata). From Majorowicz et al., 1985.



spaces; secondly, it forms conductive hydrous compounds with silicates; and thirdly, it lowers the melting point of crystalline rocks and promotes partial melting (Tuttle and Bowen, 1958; Yoder and Tilley, 1962).

Recently Gough (1986b) showed that if the continental crust contains a small amount of saline water (or brine, for brevity), then the resistive upper crust and the conductive lower crust can be explained. He suggests that in the upper crust the brine is separated in unconnected cavities due to a compressive stress field, resulting in an upper crust of relatively low conductivity; whereas in the lower crust the cavities have broken down so that the brine forms an interconnected film on the crystal surfaces, producing high conductivity.

Cox (1971) reported that serpentinized rocks typical of oceanic crustal material had shown high electrical conductivity. Experiments on serpentinized rocks from the Indian Ocean ridge by Stesky and Brace (1973) have shown that the serpentinized rocks are 3-4 orders of magnitude more conductive than unserpentinized oceanic rocks of otherwise similar compositions. De Beer et al. (1982) discussed the process of formation and the conductive mechanisms of serpentinized marine rocks in detail, and showed that such rocks in the lower crust could account for the high electrical conductivity, high magnetic susceptibility and the gravity anomalies of the Southern Cape Conductive Belt (SCCB).

As graphite is often found in fracture zones and produces conductivity anomalies, Camfield et al. (1971) proposed graphitic schists in the basement as a possible composition of the NACP conductor. Camfield and Gough (1977) reported that graphite sheets in the exposed Canadian Shield and graphitic metamorphic rocks in the Lead district of South Dakota were associated with the NACP conductor. Over a very large area of Alberta, Southern Saskatchewan, Southwestern Manitoba, and northwestern North Dakota, the sedimentary column includes deposits of salt (De Mille et al., 1964; Holter 1969). This led Camfield and Gough (1977) to propose that saline water might contribute to high conductivity of the NACP conductor. Green et al (1985) suggested an explanation that is based on the close association of the NACP conductor with the Reindeer-South Indian Lakes belt (Figure 6.3). The belt is a closed or remnant back-arc basin (Lewry et al., 1981). The high conductivity of the NACP conductor may be caused by buried slices of hydrated oceanic-type material in the lower crust (Drury and Niblett, 1980), situated within the belt and adjoining geological units. Seismic low-velocity zones across the Reindeer-South Indian Lakes belt (profiles E and H) support this explanation (Green et al., 1985).

It is possible that during the Precambrian a narrow sea extended from the ancestor of the Pacific Ocean through Idaho, British Columbia, and Alberta into Saskatchewan (Kanasewich, 1968) and that during a collision or subduction

of two plates the part of oceanic crust became trapped and kept its water-rich character. It has been known that the Cambrian sea transgressed eastward from the Cordillera through to the exposed Canadian Shield (van Hees and North, 1965). So a buried segment of the oceanic crust containing brine and, probably, serpentinized rocks is a possible cause of the high conductivity in the SABC conductor. These speculations, however are difficult to test in a conductor that remains at lower crustal depths throughout its length.

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# APPENDIX I

## Geographic coordinates of stations in 1983 array

|   |                            | Degree N | Degree W |
|---|----------------------------|----------|----------|
| 1 | CRS (CRESTON)              | 49.12    | 116.58   |
| 1 | YAK (YAHK)                 | 49.07    | 116.07   |
| 1 | KAS (KOOKANUSA)            | 49.05    | 115.03   |
| 1 | CAR (CARDSTON)             | 49.23    | 113.25   |
| 2 | FAU (FAUQUIER)             | 49.90    | 118.01   |
| 2 | NED (NEW DENVER)           | 49.98    | 117.37   |
| 2 | CRA (CRAWFORD BAY)         | 49.67    | 116.82   |
| 2 | SMR (ST MARY RIVER)        | 49.67    | 116.30   |
| 2 | FTS (FORT STEELE)          | 49.62    | 115.63   |
| 2 | ELK (ELK RIVER)            | 49.85    | 114.87   |
| 2 | PIN (PINCHER)              | 49.48    | 113.95   |
| 3 | CPH (CAPE HORN)            | 50.35    | 117.88   |
| 3 | TOB (TOBY CREEK)           | 50.35    | 116.43   |
| 3 | WIR (WILMER)               | 50.55    | 116.07   |
| 3 | CNF (CANAL FLATS)          | 50.15    | 115.82   |
| 3 | NWR (NORTH WHITE RIVER)    | 50.23    | 115.28   |
| 3 | STR (STIMPSON CREEK)       | 50.28    | 114.25   |
| 3 | CAH (CLARESHOLM)           | 50.03    | 113.72   |
| 3 | OMR (OLDMAN RIVER)         | 49.87    | 112.62   |
| 4 | REL (REVELSTOKE)           | 50.97    | 118.13   |
| 4 | KOR (KOOTENAY R. CROSSINE) | 50.88    | 116.05   |
| 4 | HIF (HILLSDALE FLATS)      | 51.25    | 115.78   |
| 4 | HWR (HIGHWOOD RIVER)       | 50.78    | 113.83   |
| 4 | MLO (MILO)                 | 50.57    | 112.82   |
| 5 | TOM (THOMPSON)             | 52.03    | 116.67   |
| 5 | WFL (WATERFOWL LAKE)       | 51.83    | 116.60   |
| 5 | RED (RED DEER RIVER)       | 51.75    | 115.55   |
| 5 | CRM (CREMONA)              | 51.60    | 114.35   |
| 5 | DRU (DRUMHELLER)           | 51.50    | 112.83   |

## APPENDIX II

### Fourier transform (FT)

The Fourier transform used for contoured maps of Fourier transform parameters is summarized here.

Any function  $f(t)$  which is piecewise smooth over  $(0, T)$  and periodic with  $T$  may be expanded as

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{+in\omega_0 t} \quad (II.1)$$

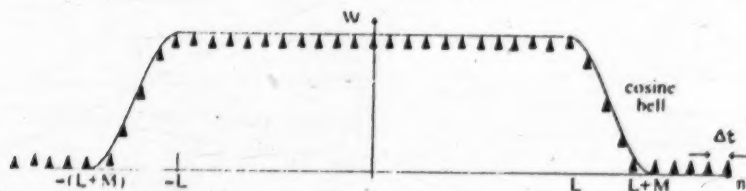


FIGURE II.1 A data window used to taper a discrete function

where  $\omega_0 = \frac{2\pi}{T}$  is the fundamental frequency of  $f(t)$ .

The coefficients  $F_k$  can be evaluated by using the orthogonal property of the functions  $e^{+in\omega_0 t}$  over the interval  $(0, T)$ . To determine  $F_k$  observe that

$$\begin{aligned} \int_0^T f(t) e^{+ik\omega_0 t} dt &= \int_0^T \sum_{n=-\infty}^{\infty} F_n e^{+in\omega_0 t} e^{+ik\omega_0 t} dt \\ &= \sum_{n=-\infty}^{\infty} F_n \int_0^T e^{+i(k-n)\omega_0 t} dt \end{aligned}$$

$$\text{Since } \int_0^T e^{-i(k-n)\omega_0 t} dt = \begin{cases} T & \text{for } k=n \\ 0 & \text{for } k \neq n, \end{cases}$$

$$F_k = \frac{1}{T} \int_0^T f(t) e^{-ik\omega_k t} dt \quad (II.2)$$

The discrete forms of (II.1) and (II.2) can be written

as

$$f_n = \sum_{m=0}^{N-1} F_m e^{+2\pi i m n / N} \quad n = 0, 1, \dots, N-1 \quad (II.3)$$

and

$$F_m = \frac{1}{N} \sum_{n=0}^{N-1} f_n e^{-2\pi i m n / N} \quad m = 0, 1, \dots, N-1 \quad (II.4)$$

where  $N$  is the total number of points sampled at intervals of  $\Delta t$  seconds and the frequency in radians is

$$\omega_m = 2\pi / (N\Delta t) \quad m = 0, 1, \dots, N/2 \quad (II.5)$$

and period in seconds is

$$T_m = 2\pi / \omega_m = N\Delta t / m \quad m = 0, 1, \dots, N/2 \quad (II.6)$$

Since  $f_n$  and  $F_m$  are complex, we can write

$$F_m = a_m + ib_m \quad (II.7)$$

The amplitude spectrum is defined as

$$|F_m| = (a_m^2 + b_m^2)^{1/2} \quad (II.8)$$

and the phase spectrum is defined as

$$\Phi_m = \tan^{-1}(b_m/a_m) \quad (II.9)$$

A magnetovariation field event is chosen within a time interval  $T$ , at the end of which each of the three components  $x(t)$ ,  $y(t)$  and  $z(t)$  returns to near its initial value. A straight line trend drawn between the first and last point is removed. Any large discontinuity may occur between the values of the function at the beginning and end of the vector of data will introduce high frequency components into



Fourier transform. To avoid this, the first and last 10 percent of data are tapered to the mean value, which is usually zero. The taper consists of a pair of cosine bells. For  $N$  data points the  $n^{\text{th}}$  component is multiplied by

$$W_n = \begin{cases} (1 + \cos \frac{\pi(n+L)}{M})/2 & -(L+M) < n < -L \\ 1 & -L \leq n \leq L \\ (1 + \cos \frac{\pi(n-L)}{M})/2 & L < n < L+M \end{cases} \quad (\text{II.10})$$

Figure II.1 illustrates this taper, where  $N = 2(L+M)$  and  $M = N/10$ .

The subroutine RHARM (Cooley-Tukey Fast Fourier transform algorithm) in the IBM Scientific Subroutine package (\*SSPLIB) was used to compute Fourier spectra. The subroutine RHARM requires the number of data points to be an integral power of 2; zeros are added to make  $2^{10}$  data points.